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X-ray focusing by the system of refractive lens(es) placed inside asymmetric channel-cut crystals

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An X-ray one-dimensionally focusing system, a refracting–diffracting lens (RDL), composed of Bragg double-asymmetric-reflecting two-crystal plane parallel plates and a double-concave cylindrical parabolic lens placed in the gap between the plates is described. It is shown that the focal length of the RDL is equal to the focal distance of the separate lens multiplied by the square of the asymmetry factor. One can obtain RDLs with different focal lengths for certain applications. Using the point-source function of dynamic diffraction, as well as the Green function in a vacuum with parabolic approximation, an expression for the double-diffracted beam amplitude for an arbitrary incident wave is presented. Focusing of the plane incident wave and imaging of a point source are studied. The cases of non-absorptive and absorptive lenses are discussed. The intensity distribution in the focusing plane and on the focusing line, and its dependence on wavelength, deviation from the Bragg angle and magnification is studied. Geometrical optical considerations are also given. RDLs can be applied to focus radiation from both laboratory and synchrotron X-ray sources, for X-ray imaging of objects, and for obtaining high-intensity beams. RDLs can also be applied in X-ray astronomy.

Keywords: X-ray dynamical diffraction; X-ray refraction; X-ray optics; X-ray imaging; X-ray astronomy.

1. Introduction

Nowadays, X-ray focusing is considered to be an important aspect of studies in physics. Focusing is necessary for the imaging of objects (including those not transparent in the visible spectrum), the creation of X-ray microscopes, for X-ray astronomy and other physical studies. Lenses can be used as the focusing element. However, the focal distances of X-ray lenses are very large (refractive indices are very close to 1), and their application meets principle difficulties. It becomes essential to have systems which have focusing distances (of 1 m order) appropriate for application. An X-ray focusing system has been suggested (Grigoryan *et al.*, 2004) that consists of $(+n, -n)$ asymmetric-reflecting two-crystal plane parallel plates with a double-concave cylindrical parabolic lens (refractive index < 1) placed in the gap between the plates (Fig. 1). The axis of the parabolic cylinder is perpendicular to the diffraction plane, and the beam diffracted from the first plate in the diffraction plane falls perpendicularly onto the lens. This system is called a refractive–diffractive lens (RDL). Grigoryan *et al.* (2004) show the focusing principle capability of a double-diffracted beam with its trajectory approximation,

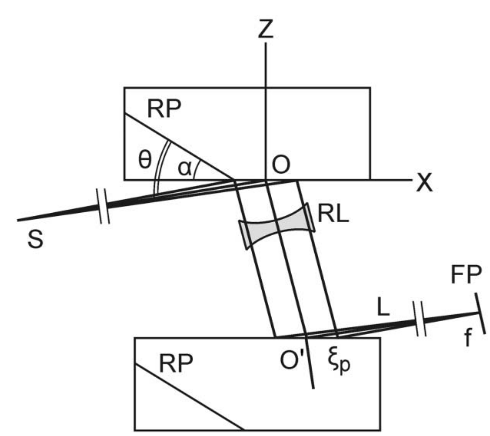

Figure 1
The general focusing scheme in the RDL. S: X-ray point source; Ox, Oz: coordinate axes; RP: reflecting planes; θ: glancing angle formed by the reflecting planes and the incident beam; α: angle between the reflecting planes and the entrance surface; RL: refractive cylindrical parabolic lens with variables L and ξ_p and their directions of increase, parallel and perpendicular to the double-diffracted beam, respectively, and O' their origin; FP: focal plane perpendicular to the double-diffracted beam; f: focus.

as well as determine the focal distance. This is equal to the focal distance of the separate lens multiplied by the square of the asymmetry factor. In the case where the asymmetry factor is 0.05, the focal distance of the separate lens in the RDL decreases by a factor of 400; in this case the RDL is equivalent to a one-dimensional focusing compound X-ray lens consisting of 400 lenses (Lengeler *et al.*, 1999). For a point source placed at a distance L_s, the image distance L_f in a RDL is determined using the same formula as for standard optical lenses, $1/L_s + 1/L_f = 1/F$, but the focal distance of a RDL, F, is equal to $F_0 b^2$, where $F_0 = R/2\delta$ is the focal distance of the individual lens, R is the curvature radius of the lens at the apex, δ is the material decrement of the lens and b is the asymmetry factor. Since it is possible to vary the reflection parameters, particularly the asymmetry factor, the radius of the lens and the material used, RDLs with various focal distances can be obtained. The advantage of RDLs over focusing by flat or bent crystals or by other types of Bragg-focusing elements (Hrdý *et al.*, 2006) is that the diffraction and focusing are accomplished through separate elements realising these phenomena. In the future this idea may serve as a basis for calculation of focusing elements of other types. In particular, it would be interesting to study the cases where a zone plate, focusing mirror, or plane or curved crystals are placed in the gap instead of the lens and to calculate the obtained focal distances depending on the asymmetry factor. Practically, the combination of double asymmetric reflection and focusing is important at this point. The reflection can be achieved not only by the Bragg-diffracting method but also by using other methods of double asymmetric reflection. It is not inconceivable that the use of a double asymmetric reflection and focusing combination may also be applicable for focusing electromagnetic radiation of other wavelengths (particularly in the visual spectrum) or other kinds of waves, such as neutrons.

The purpose of the current work is, on the basis of the dynamic diffraction theory, to obtain the general expression for the double-diffracted X-ray beam amplitude of a RDL for the case of an arbitrary incident wave (wave optical consideration). It is also aimed to study the intensity distribution on the focusing line and in the focusing plane for a plane wave and for a point source, to determine the longitudinal and transverse sizes of the focal spot, as well as to study the dependence of the focusing phenomenon on the wavelength, the Bragg angle deviation and the magnification. The cases of absorptive and non-absorptive lenses are considered. Based on the wave optical method the relations between the distances of the source to the RDL, L_s, the RDL to the focusing point, L_f, and the focal distance, F, are given. It is shown that F is equal to $F_0 b^2$. The resolutions, focal distances of the RDL and the separate cylindrical lens are calculated and compared. Geometric optical consideration of the RDL is given in §2. Here, in addition to the results obtained by Grigoryan *et al.* (2004), new results are presented concerning the magnification and polychromatic character of the incident beam. Similar to the wave optical consideration, using geometric optical methods it is also shown that the focus distance in the paraxial approximation is independent of the wavelength. The influence of the lens on the Bragg reflection from the second plate is considered by geometric optical methods. The apertures of the incident beam and the lens in connection with the Bragg condition are considered as well. The transmission of the RDL is investigated and compared with the transmission of individual cylindrical lenses.

2. Geometric optical consideration of RDLs

2.1. Focal distance: lens formula and magnification

First it is worth considering the problem by geometric optical methods. Let us suppose that an X-ray polychromatic spherical-wave beam falls on the first-crystal plate from a point source S (Fig. 1), and the beam has maximum intensity for wavelength λ_m and intensity distribution by wavelengths in a small range of wavelengths ($|\Delta\lambda/\lambda_m| \ll 1$). This can be a characteristic radiation line. The distance between the point source and the RDL is L_s. If θ is the glancing angle formed by the beam and the reflecting planes at the origin O (the center of the beam), then at any point with coordinate x on the entrance surface of the first plate the incident ray forms a glancing angle with the reflecting planes of

$$\theta(x) = \theta - x\sin(\theta - \alpha)/L_s. \quad (1)$$

Here α is the angle which forms where the reflecting plane meets the surface of the plate (Fig. 1). For any λ the deviation from the exact Bragg angle at any point x is $\Delta\theta(x,\lambda) = \theta(x) - \theta_0(\lambda)$, where $\theta_0(\lambda)$ is the exact Bragg angle for wavelength λ. However, the glancing angles for all wavelengths are the same at the same point and are determined by (1). Let us determine the directions of the Bragg-reflected rays at each point x. Using the continuity condition of the tangential component of the reflected wavevector at the point x, one can write

$$K^e_{hx}(x) = K_{0x}(x) + h_x. \quad (2)$$

Here $\mathbf{K}^e_h(x)$ is the wavevector of the Bragg-reflected wave at the point x, $\mathbf{K}_0(x)$ is the wavevector of the incident wave at the same point, $\mathbf{h}$ is the reciprocal lattice vector satisfying the reflection condition and has the components $h_x = -|\mathbf{h}|\sin\alpha$, $h_z = -|\mathbf{h}|\cos\alpha$, where $|\mathbf{h}| = 2k\sin\theta_0(\lambda)$, $k = 2\pi/\lambda$. $|\mathbf{h}|$ is independent of the wavelength λ. It is clear that

$$K_{0x}(x) = k\cos[\theta(x) - \alpha] \simeq k\cos[\theta_0(\lambda) - \alpha] - k\gamma_0\Delta\theta(x,\lambda), \quad (3)$$

where $\gamma_0 = \sin(\theta_0 - \alpha)$. On the other hand,

$$K^e_{hx}(x) = k\cos\big[\theta_h(x,\lambda) + \alpha\big], \quad (4)$$

where $\theta_h(x,\lambda)$ denotes the glancing angle of the Bragg-reflected ray at the point x. Using (2), (3) and (4),

$$k\cos\big[\theta_h(x,\lambda) + \alpha\big] \simeq k\cos\big[\theta_0(\lambda) + \alpha\big] - k\gamma_0\Delta\theta(x,\lambda)$$

is obtained. Determining $\Delta\theta_h(x,\lambda) = \theta_h(x,\lambda) - \theta_0(\lambda)$ and inserting it into the above equation, the following relation is obtained,

$$\Delta\theta_h(x,\lambda) = b\Delta\theta(x,\lambda), \quad (5)$$

where $b = \gamma_0/\gamma_h$ is the asymmetry factor, $\gamma_h = \sin(\theta_0 + \alpha)$, and $\Delta\theta_h(x, \lambda)$ is a function of λ. This means that at each point on the entrance surface of the first plate the directions of the reflected rays differ for various λ. It follows from (5) that

$$\theta_h(x, \lambda) = \theta_0(\lambda) + b\Delta\theta(x, \lambda) = \theta(x) + (b-1)\Delta\theta(x, \lambda). \quad (6)$$

It is assumed that the lens is perpendicular to the direction which forms the glancing angle ($\theta + \alpha$) with the entrance surface of the first plate (Fig. 1). The direction determined by the glancing angle ($\theta + \alpha$) does not depend on wavelength. Then it is obvious from (6) that the ray reflected at the point $x = 0$ (at the origin O) is deviated from the center of the lens by $D_h\Delta\bar{\theta}_0$, where D_h is the characteristic size of the gap along the direction of the reflected rays and $\Delta\bar{\theta}_0$ is an average deviation from the exact Bragg condition. If $\Delta\bar{\theta}_0 \simeq 10^{-4}$ and $D_h \simeq$ 10 mm then the deviation is 1 µm. This deviation must be neglected as the diffracted rays have geometrical sizes much greater than 1 µm. At the other points x these deviations can be neglected for all wavelengths. Below it becomes clear that these deviations are negligible because the difference between the refraction angles in the lens at the points x and $x + D_h\Delta\bar{\theta}_0$ is also negligible. Therefore, all the rays pass through the lens having the same parameter x, which they have on the entrance surface of the first plate. Coordinate $x\sin(\theta + \alpha) \simeq x\gamma_h$ on the entrance surface of the lens corresponds to the parameter x on the entrance surface of the first plate. The formula for the surface of the lens is $x^2{\gamma_h}^2/2R$ + constant, where R is the radius of the parabolic lens at the apex. The derivative of this formula by the variable $x\gamma_h$ gives the tangent of the angle which forms the two normals to the parabola at the points O and $x\gamma_h$. Therefore,

$$\tan\varphi = x\gamma_h/R. \quad (7)$$

According to Snell's law,

$$\sin\varphi/\sin(\varphi + \Delta\varphi) = n = 1 - \delta, \quad (8)$$

where $\varphi + \Delta\varphi$ gives the angle of the refracted ray, formed with the normal to the lens surface at the point $x\gamma_h$, n is the refractive index of the lens material and δ is the decrement. It is assumed that the rays falling on the lens surface are parallel to the lens axes. The contributions of the deviations of the directions from the parallel to the axes of the lens direction in $\Delta\varphi$ are negligible. Since δ and $\Delta\varphi$ are small, a linear approximation can be used and, from (8), $\Delta\varphi = \delta\tan\varphi = x\delta\gamma_h/R$. After passing the second surface of the lens, the ray changes its direction by

$$\Delta\varphi_{\rm t} = 2x\delta\gamma_h/R. \quad (9)$$

Now, from (9) it is clear that if the shift of x is $\Delta x = D_h\Delta\bar{\theta}_0 =$ 1 µm, for $\delta \simeq 10^{-6}$, $\gamma_h \simeq 0.3R \simeq 1$ mm, the corresponding deviation of $\Delta\varphi_{\rm t}$ is $\sim 6 \times 10^{-10}$. This means that the change of x can be neglected when the reflected ray passes the distance from the entrance surface of the first plate to the entrance surface of the lens. Taking into account (6) and (9), the glancing angles of the rays falling on the surface of the second plate are determined as

$$\begin{aligned}\theta_h'(x, \lambda) &= \theta_0(\lambda) + b\Delta\theta(x, \lambda) + 2x\delta\gamma_h/R \\ &= \theta_0(\lambda) + b\Delta\theta(0, \lambda) + bx\gamma_0(1/F - 1/L_{\rm s}), \quad (10)\end{aligned}$$

where $F = F_0b^2$ and $F_0 = R/2\delta$ is the focal distance of the individual lens. For $\Delta\theta_h'(x, \lambda) = \theta_h'(x, \lambda) - \theta_0(\lambda)$ from (10) it follows

$$\Delta\theta_h'(x, \lambda) = b\Delta\theta(0, \lambda) + bx\gamma_0(1/F - 1/L_{\rm s}). \quad (11)$$

The x parameter of the rays falling on the surface of the second plate slightly differs from that falling on the second surface of the lens. This difference is $\Delta x \simeq 1$ µm and is negligible because after the Bragg reflection of the rays from the surface of the second plate this difference is $\gamma_0\Delta x \simeq$ 0.02 µm ($\gamma_0 \simeq 0.02$) in the double-diffracted beam cross section. It follows from (5) that after Bragg reflection from the second plate the deviations of the rays are

$$\begin{aligned}\Delta\theta'(x, \lambda) &= \Delta\theta_h'(x, \lambda)/b \\ &= \Delta\theta(0, \lambda) + x\gamma_0(1/F - 1/L_{\rm s}). \quad (12)\end{aligned}$$

Since $\theta_0(\lambda) + \Delta\theta(0, \lambda) = \theta$ is independent of λ, then

$$\theta'(x) = \theta_0(\lambda) + \Delta\theta'(x, \lambda) \quad (13)$$

does not depend on wavelength. Here x is calculated from the point O' (Fig. 1). The rays are reflected from the second plate at the glancing angles (13) to the reflecting planes and at the glancing angles $\theta'(x) - \alpha$ to the entrance surface of the second plate. The central reflected ray and the ray reflected at the point x form the angle $\Delta\theta'(x, \lambda) - \Delta\theta'(0, \lambda) = x\gamma_0(1/F - 1/L_{\rm s})$ and are intersected on the central line at the distance $L(x)$ from the point O' determined from the expression

$$\begin{aligned}x\gamma_0/L(x) &= \Delta\theta'(x, \lambda) - \Delta\theta'(0, \lambda) \\ &= x\gamma_0(1/F - 1/L_{\rm s}). \quad (14)\end{aligned}$$

This relation gives the focusing distance $L = L_{\rm f}$. It follows from (14) that $L_{\rm f}$ is independent of x and λ,

$$1/L_{\rm s} + 1/L_{\rm f} = 1/F. \quad (15)$$

As follows from (13)–(15), the coordinates ($\xi_{\rm p} = 0$, $L = L_{\rm f}$) (Fig. 1) of the double-diffracted beam focus point do not depend on wavelength, *i.e.* the image of a point chromatic source has no chromatic aberrations and is still a point.

If $R \simeq 1$ mm, $\delta \simeq 10^{-6}$, $b \simeq 0.05$, then $F_0 \simeq 500$ m, $F \simeq 1$ m and the focal distance of the RDL is equivalent to the focal distance of a compound lens containing 400 cylindrical parabolic lenses. As follows from (15), when $L_{\rm s} \rightarrow \infty$, then $L_{\rm f} = F$. This is the case of the incident plane wave. When $L_{\rm s} = F$, then $L_{\rm f} \rightarrow \infty$, *i.e.* the source is placed at the focal distance and a plane wave is formed by the RDL. This is obvious from (12) too. In the case $L_{\rm s} = F$, $\Delta\theta'(x, \lambda) = \Delta\theta(0, \lambda)$ is independent of x, *i.e.* all the rays reflected from the second plate are parallel to the central ray.

Now let us consider another point source S_1 which is at the same distance $L_{\rm s}$ and deviated from the first point source S perpendicular to the propagation direction of the incident wave by $\Delta\xi_{\rm s}$. The beam emitted from this point source falls on the RDL at the glancing angle θ_1 to the reflecting planes. In paraxial approximation the perpendicularity of the lens to the reflecting rays from this point source is also true. All formulae obtained for the first point source are also true for the second one. However, the rays of the second point source are focused on the central line of the double-reflected beam of the second source, *i.e.* the focusing line for the second point source forms angle $(\theta_1 - \theta)$ with the focusing line of the first point source. Therefore, the reversed image is formed. The images are formed at the same distance, but are deviated by

$$\Delta\xi_{\rm f} = L_{\rm f}(\theta_1 - \theta) = -\Delta\xi_{\rm s} L_{\rm f}/L_{\rm s}. \tag{16}$$

Hence the magnification is determined as

$$M = L_{\rm f}/L_{\rm s}. \tag{17}$$

It can be seen from (15) that if $F \le L_{\rm s} \le 2F$, then $L_{\rm f} \ge 2F$ and $M \ge 1$. In the case where $L_{\rm s} > 2F$ it follows that $F \le L_{\rm f} < 2F$ and $M < 1$.

2.2. Amplitudes of diffracted beams: influence of the lens on the Bragg condition

Now let us consider the amplitudes of the rays reflected from the first and the second crystals. At the point x the deviation from the exact Bragg angle is given by (1). For any deviation the amplitude coefficient of reflection of a Bragg-reflected ray (Pinsker, 1982) is well known,

$$\Gamma_1(x,\lambda) = -\left(\chi_h/\chi_{\bar h}\right)^{1/2}(\gamma_0/\gamma_h)^{1/2}\sigma\Big/\Big(k\Delta\theta(x,\lambda)\gamma_0 + \sigma_0 + \Big\{\big[k\Delta\theta(x,\lambda)\gamma_0+\sigma_0\big]^2 - \sigma^2\Big\}^{1/2}\Big), \tag{18}$$

where $\sigma^2 = k^2\chi_h\chi_{\bar h}\gamma_0\gamma_h/\sin^2 2\theta_0$, $\sigma_0 = k\chi_0\cos\alpha/2\cos\theta_0$ and $\chi_0, \chi_h, \chi_{\bar h}$ are the crystal dielectric susceptibility Fourier components corresponding to the zero and **h** reflection. It follows from (18) that the dimension of the region on the entrance surface of the first crystal, where the Bragg reflection takes place, is determined from the condition

$$\left|\Delta\theta_{\rm c}(x,\lambda)\right| \le \left|\chi_h\right|(\gamma_h/\gamma_0)^{1/2}/\sin 2\theta_0, \tag{19}$$

where $\Delta\theta_{\rm c}(x,\lambda) = -\psi_0 + \Delta\theta(x,\lambda)$ is the deviation from the Bragg-corrected angle $\theta_{\rm c}(\lambda) = \theta_0(\lambda) + \psi_0$ and $\psi_0 = \left|\chi_0\right|(1+\gamma_h/\gamma_0)/2\sin 2\theta_0$. Let us consider the case when $\Delta\theta_{\rm c}(0,\lambda) = 0$, *i.e.* the beam falls at the Bragg-corrected angle for λ. It can be $\lambda_{\rm m}$. In this case $\Delta\theta_{\rm c}(x,\lambda_{\rm m}) = -x\gamma_0/L_{\rm s}$. Using (19) the following estimation is obtained,

$$\left|x\gamma_0/L_{\rm s}\right| \le \Delta\psi, \tag{20}$$

where $\Delta\psi$ is the right-hand side of (19). Assuming that $|\chi_h| \simeq 1.9\times10^{-6}$, $\sin 2\theta_0 \simeq 0.36$, $\gamma_0 \simeq 0.017$, $\gamma_h \simeq 0.35$, $b \simeq 0.05$, $L_{\rm s} \simeq 1$ m, $\Delta\psi = 2.3\times10^{-5}$, $|x| \simeq 1.3$ mm is obtained. If the projection of the falling beam on the entrance surface of the first plate is $2R_{0x} = 6$ mm, then it is equal to more than two whole rocking curves ($4\Delta\psi$) in real space. The transverse size of the incidence beam is $6\gamma_0$ (mm) $\simeq 105$ µm. The transverse size of the Bragg-reflected beam is $6\gamma_h$ (mm) $\simeq 2$ mm. The aperture $2R_0$ of the lens must be $2R_0 \ge 2$ mm.

Similarly to (18), the amplitude reflection coefficient of the reflected beam from the second plate is

$$\Gamma_2(x,\lambda) = -\left(\chi_{\bar h}/\chi_h\right)^{1/2}(\gamma_h/\gamma_0)^{1/2}\sigma\Big/\Big(k\Delta\theta_h'(x,\lambda)\gamma_h + \sigma_0 + \Big\{\big[k\Delta\theta_h'(x,\lambda)\gamma_h+\sigma_0\big]^2 - \sigma^2\Big\}^{1/2}\Big). \tag{21}$$

Now the influence of the lens on the reflection condition can be estimated. In the case where the wave component corresponding to $\lambda_{\rm m}$ falls at the Bragg-corrected angle, as is seen from (20) the component of the reflected beam for $\lambda_{\rm m}$ is strongly reflected at the point x if

$$\left|x\gamma_0(1/F - 1/L_{\rm s})\right| = \left|x\gamma_0/L_{\rm f}\right| \le \Delta\psi. \tag{22}$$

This estimation must be considered in combination with (20), which is fulfilled. If $L_{\rm f} \ge L_{\rm s}$ ($M \ge 1$) then (22) is fulfilled in the same region of x as (20). In the case when $L_{\rm f} < L_{\rm s}$ ($M < 1$) the region of strong reflection on the surface of the first plate increases, and the region of strong reflection on the surface of the second plate decreases. From (22) it is obvious that the minimal value $|x_{\min}| = F\Delta\psi/\gamma_0 = 1.15$ mm of the strong reflection region on the surface of the second plate for $M < 1$ is achieved for $L_{\rm f} = F$, and the maximal value $|x_{\max}| \simeq 2.3$ mm for $M < 1$ is achieved for $L_{\rm f} \simeq 2F$. Using (18) the distances $L_{\rm s}$ for which the plane-wave approximation is valid can also be estimated,

$$\left|\Delta\theta_{\rm c}(x,\lambda)\right| \ll \Delta\psi. \tag{23}$$

From (23) it follows that the plane-wave approximation is valid when

$$L_{\rm s} \gg |x|\gamma_0/\Delta\psi. \tag{24}$$

Taking $|x| = 3$ mm from (24), then $L_{\rm s} \gg 2.6$ m is found. If a plane parallel beam falls at the appropriate angle, the whole region on the first plate (6 mm) is a region of strong reflection, while the second plate is strongly reflected in the region given by the estimation (22), *i.e.* about $2|x_{\min}| = 2.3$ mm. Combining (18) and (21) the whole reflection amplitude at any point x can be found as

$$\Gamma(x,\lambda) = \Gamma_1(x,\lambda)\Gamma_2(x,\lambda). \tag{25}$$

It is easy to calculate the reflection amplitude at any point using (25) and to obtain the graphic of $|\Gamma|^2$ for any wavelength, incident angle and distance $L_{\rm s}$. For each case the region for which the intensity value of the double-reflected beam on the surface of the second plate does not vanish will be seen in the graphic.

It is interesting to note that the formula (25) is sufficient for calculating the double-diffracted field amplitude in a vacuum after reflection from the second plate. For this, it is necessary to write the phase of (25) and find the amplitude of the double-diffracted beam at any distance from the RDL in a vacuum using the Huyghens–Fresnel principle. In the focusing plane this operation gives the same result as Fraunhofer diffraction at the glancing angle $(\theta - \alpha)$ on the slit with the same aperture as of the projection of the lens on the surface of the second plate and with the amplitude transmission coefficient Γ. The final result for the amplitude $E^{e}_{0h\rm f}$ of the double-diffracted wave on the focusing plane is

$$E^{e}_{0hf} = \exp(-i\pi/4)(F/2\pi)^{1/2}\big[\sin(\theta-\alpha)/L_{\rm f}L_{\rm s}\big] \times \exp\big[iky^2/2(L_{\rm f}+L_{\rm s})\big]\exp(-k|\beta|T_0) \times \int_{-R_{0x}}^{R_{0x}} \Gamma(x',\lambda)\exp(-k\gamma_1 x'^2\gamma_0^2/2F)\exp(-ikxx'\gamma_0^2/L_{\rm f})\,{\rm d}x'$$

where γ_1 = $|\beta/\delta|$, the refractive index of the lens is n = $1-\delta-i\beta$ and T_0 is the thickness of the lens at the apex. For the large lens the limits of integration can be taken $(-\infty, +\infty)$. The validity of this approximation will be given in §4.1. However, more accurate consideration of this problem will be carried out using wave optical methods and dynamical diffraction theory.

2.3. Transmission

The losses in RDLs are determined by absorption both in the lens and in the plates and by the Bragg strong reflecting common region of the first and the second crystals. For a lens the absorption effect is small and it can be omitted by choosing appropriate material for the lens. The absorption in the plates in the Bragg case for more reflections [the example of Si(220) (Mo $K\alpha$) is considered] is small. However, for any wavelength the losses owing to the change of the Bragg reflection condition by the lens are more important. The whole flux reflected from the RDL for each wavelength is determined by the square module of (25) multiplied by the size of the common region of the strong Bragg reflection of two crystalline plates and the lens. So, the transmission of the RDL can be determined as the ratio of the reflected beam flux on the surface of the second crystal to the flux of the Bragg reflection from the surface of the first-plate beam. It is equal to the ratio of the common Bragg-reflection region to the reflection region from the first plate and is multiplied by $|\Gamma(\bar{x},\lambda)|^2$, where $\bar{x}$ is the middle point coordinate of the common region of the Bragg reflection for the whole system. This is equal to 0.94 for the Si(220) (Mo $K\alpha$) reflection, and the transmission connected only with absorption in the plates is $0.94 \simeq 1$. It is more difficult to estimate the ratio of the size of the common reflecting region to the size of the reflection region on the surface of the first plate. This estimation has been given [see (20), (22)] for the wavelength for which the deviation from the Bragg-corrected angle is zero at the origin. Let us take another λ. It is true to see from (1) and (11) that

$$\begin{aligned}\Delta\theta_{\rm c}(x,\lambda) &= -\Delta\lambda\tan\theta/\lambda - x\gamma_0/L_{\rm s},\\ \Delta\theta'_{h{\rm c}}(x,\lambda) &= -b\Delta\lambda\tan\theta/\lambda + bx\gamma_0/L_{\rm f},\end{aligned} \qquad (26)$$

where $\Delta\theta_{\rm c}(x,\lambda)$ and $\Delta\theta'_{h{\rm c}}(x,\lambda)$ are the deviations from the corrected Bragg angle at the point x for the wavelength λ for the first and the second crystals, respectively. For the first crystal, x is calculated from O, and, for the second crystal, from O', $\Delta\lambda = \lambda-\lambda_{\rm m}$ and $\Delta\theta_{\rm c}(0,\lambda_{\rm m}) = 0$. It is seen from (26) that the regions of Bragg reflection are concentrated around the points

$$\begin{aligned}x_0(\lambda) &= -(L_{\rm s}/\gamma_0)\Delta\lambda\tan\theta/\lambda,\\ x_h(\lambda) &= (L_{\rm f}/\gamma_0)\Delta\lambda\tan\theta/\lambda\end{aligned} \qquad (27)$$

on the surfaces of the first and the second crystal, respectively. The intervals around the points $x_0(\lambda)$ and $x_h(\lambda)$, where the Bragg reflection takes place, are

$$\begin{aligned}&\big[x_0(\lambda)-(L_{\rm s}/\gamma_0)\Delta\psi,\, x_0(\lambda)+(L_{\rm s}/\gamma_0)\Delta\psi\big],\\ &\big[x_h(\lambda)-(L_{\rm f}/\gamma_0)\Delta\psi,\, x_h(\lambda)+(L_{\rm f}/\gamma_0)\Delta\psi\big].\end{aligned} \qquad (28)$$

These two intervals have no common points when $|\Delta\lambda\tan\theta/\lambda| > \Delta\psi$. Therefore, the components with the wavelengths

$$\big|\Delta\lambda\tan\theta/\lambda\big| < \Delta\psi \qquad (29)$$

are effectively reflected.

The following cases can be derived from (27)–(29):

(a) The interval of x_h is included in the interval of x_0. In this case

$$L_{\rm s} \geq 2F/(1-\Lambda/\Delta\psi), \qquad (30)$$

where $\Lambda = |\Delta\lambda|\tan\theta/\lambda$. Obviously the size of the common region of the Bragg reflection is $2(L_{\rm f}/\gamma_0)\Delta\psi$. If the interval around x_0 includes the region of the first crystal $2R_{0x} = 2R_0/\gamma_h$, then the transmission $\Sigma = L_{\rm f}/L_{\rm s} \leq 1$. Equality to 1 corresponds to the case $[(\Lambda = 0), (L_{\rm s} = L_{\rm f} = 2F)]$. If the interval of x_0 is larger than the aperture $2R_{0x}$, but the interval of x_h is smaller than $2R_{0x}$, then $\Sigma = L_{\rm f}\Delta\psi/\gamma_0 R_{0x} \leq 1$. If these two intervals are larger than $2R_{0x}$, then $\Sigma = 1$.

(b) The interval of x_0 is included in the interval of x_h. This case is realised when

$$F \leq L_{\rm s} \leq 2F/(1+\Lambda/\Delta\psi). \qquad (31)$$

The dimensions of the common Bragg-reflecting region is equal to $2L_{\rm s}\Delta\psi/\gamma_0$. $\Sigma = 1$ in this case.

(c) The intervals of x_0 and x_h are particularly intersected. This case is realised if $2F/(1+\Lambda/\Delta\psi) < L_{\rm s} < 2F/(1-\Lambda/\Delta\psi)$. The dimension of the common Bragg-reflection region is $(L_{\rm s}+L_{\rm f})(\Delta\psi-\Lambda)/\gamma_0$. If $L_{\rm s}\Delta\psi/\gamma_0 < R_{0x}$, then $\Sigma = (L_{\rm s}+L_{\rm f})(1-\Lambda/\Delta\psi)/2L_{\rm s} \leq 1$. If the two intervals of x_0 and x_h are larger than $2R_{0x}$, then $\Sigma = 1$. $\Sigma = 1$ is also the case when the interval of x_h is included in the aperture but the interval of x_0 includes the aperture $2R_{0x}$. In comparison with the individual lens it must be noted that for a non-absorbing lens the losses are zero, *i.e.* $\Sigma = 1$.

If the absorption losses in the lens are taken into account, the transmission is determined as

$$\Sigma(\Delta\lambda, L_{\rm s}) = \exp\big(-2k|\beta|T_0\big) \times \int_{-R_{0x}}^{R_{0x}} |\Gamma|^2\exp(-k\gamma_1x^2\gamma_0^2/F)\,{\rm d}x\,\gamma_0 \Big/ \int_{-R_{0x}}^{R_{0x}} \big|\Gamma_1\big|^2\,{\rm d}x\,\gamma_h.$$

If D_2 and D_1 are the sizes of the regions of the common Bragg reflection on the surface of the second crystal and the Bragg reflection on the surface of the first crystal, respectively, and at the same time the absorption in the lens is small and the reflection coefficients are close to 1, then transmission takes the following form,

$$\Sigma(\Delta\lambda, L_s) = \exp\left(-2k|\beta|T_0\right) \int_{D2} \exp(-k\gamma_1 x^2 \gamma_0^2/F)\,dx/D_1 \simeq D_2(\Delta\lambda, L_s)/D_1(\Delta\lambda, L_s).$$

The analysis of the situation where the absorption in the lens is neglected is given by (22) for the cases (*a*)–(*c*). However, the transmission of the RDL can be introduced as the ratio of the reflected flux to the flux of the incidence wave, *i.e.*

$$\Sigma_t(\Delta\lambda, L_s) = \exp\left(-2k|\beta|T_0\right) \times \int_{-R_{0x}}^{R_{0x}} |\Gamma|^2 \exp(-k\gamma_1 x^2 \gamma_0^2/F)\,dx/(2R_{0x}).$$

For the individual absorbing lens the transmission is defined as

$$\Sigma_l = \exp(-2k|\beta|T_0) \int_{-R_0}^{R_0} \exp(-k\gamma_1 x^2/F_0)\,dx/(2R_0).$$

Here T_0 is the thickness of the lens at the apex. When the absorption is neglected, Σ_l is equal to 1. In Figs. 2 and 3, $\Sigma_t(\Delta\lambda, L_s)$ dependences are shown for the cases $\Delta\lambda = 0$ and $L_s = 3F/2$ for lenses made of beryllium and silicon, with R = 1 mm, R_0 = 3 mm, T_0 = 0.1 mm, b = 0.05. For beryllium F = 1.14 m and for silicon F = 0.8 m. For the lens made of beryllium Σ_l = 0.98, and Σ_l = 0.56 for that made of silicon. The Si(220) Mo $K\alpha$ reflection is used. The refractive indices of beryllium and silicon are given in §4.1 and §4.2, respectively. As can be seen from Figs. 2 and 3, the transmission of the RDL can be close to 1, and $\Sigma(\Delta\lambda, L_s) \geq \Sigma_t(\Delta\lambda, L_s)$. Note that for the compound lens the transmission can be up to 0.3 (Lengeler *et al.*, 1999) if the number of lenses is ∼40 and R_0 = 450 µm, R = 200 µm. The advantage of the RDL is the small focal distance when a lens is used, and the curvature radius and the aperture of the lens can be 1–10 mm. The compound lens with 400 lenses and R = 1 mm, R_0 = 1 mm equivalent to a RDL with b = 0.05 will have a longitudinal size of more than 0.4 m and negligible transmission, thus it will be practically useless.

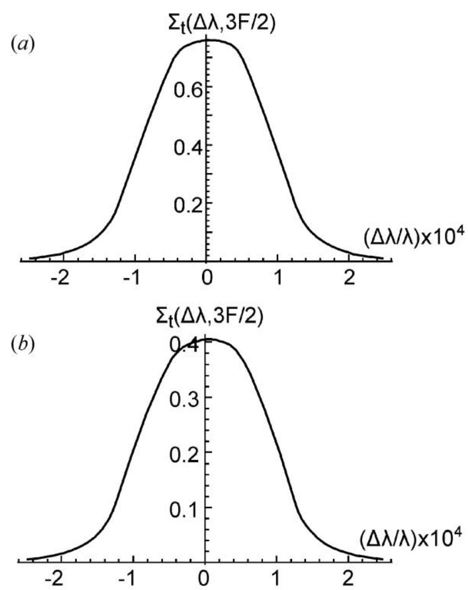

Figure 3
Transmission of the RDL and its dependence on $\Delta\lambda$ for $L_s = 3F/2$. (*a*) Lens made of beryllium, F = 1.14 m; (*b*) lens made of silicon, F = 0.8 m.

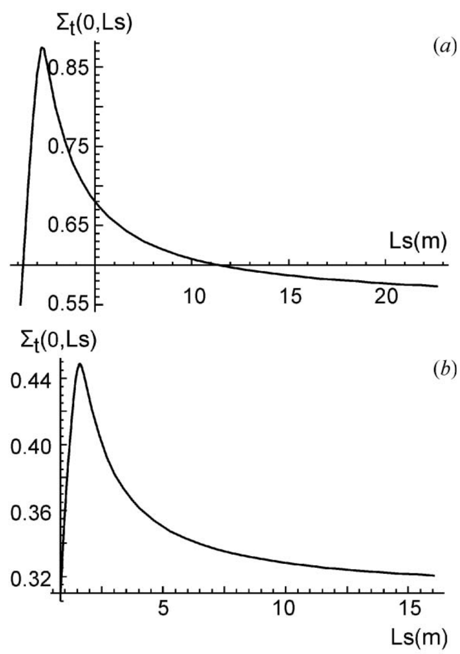

Figure 2
Transmission of the RDL and its dependence on L_s for $\Delta\lambda = 0$. (*a*) Lens made of beryllium, F = 1.14 m; (*b*) lens made of silicon, F = 0.8 m. $F \leq L_s \leq 20F$.

In summary, one-dimensional focusing can be achieved with a RDL. It is clear that for obtaining two-dimensional focusing it is necessary to use two RDLs. One of them will focus in the meridional plane and the other, placed after the first, in the plane perpendicular to the meridional plane. The vector of diffraction of the first RDL lies in the plane ($\mathbf{K}$, $\mathbf{e}_z$), the vector of diffraction of the second RDL $\mathbf{h}_2$ lies in the plane ($\mathbf{K}$, $\mathbf{e}_y$), where $\mathbf{e}_y$ is the unit vector perpendicular to the plane XOZ, and $\mathbf{K}$ is the wavevector of the incidence beam. If the asymmetry factors of these two RDLs are the same, two-dimensional point focusing is achieved, and the formula of that lens is the same as (15), with the same F. At the distance L_f the stigmatic image of a point source is formed. The calculation of (15) for two RDLs can be carried out using geometric optical methods, as represented in this section. The formulae (16) and (17) for magnification stay unchanged. In the current work, wave theory is given only for one-dimensional focusing, since the wave optical consideration of two-dimensional focusing with two RDLs is more complicated.

3. Wave optical consideration: derivation of the general formula of double-diffracted beam amplitude

Consideration of the RDL by wave optical methods is necessary. The formulae for the intensity distribution in the focusing plane and on the focusing line, the longitudinal and transverse sizes of the focal spot, and for the resolution of the lens and object imaging can be correctly obtained only by using wave optical methods. First the wave optical consideration of this problem for an arbitrary incident wave is given. Then the main formula obtained for the amplitude of the double-diffracted beam is applied for the case of an incident plane wave and for the case of point-source imaging.

The electric field of an arbitrary non-polarized incident wave is described as $\mathbf{E}^i(\omega, \mathbf{r})\exp(i\mathbf{K}\mathbf{r})$, where $\mathbf{E}^i$ is the slowly varying amplitude, and the exponent space period is of the order of interatomic spacing, and ω is the frequency corresponding to wavelength λ. $\mathbf{K}$ are wavevectors of different lengths, $\mathbf{K}^2 = (2\pi/\lambda)^2$, and $\mathbf{K}$ corresponding to different wavelengths have the same direction. The glancing angle formed by $\mathbf{K}$ and the entrance surface of the first crystal is $(\theta - \alpha)$, where θ is the glancing angle formed by $\mathbf{K}$ and the reflecting atomic planes, and α is the angle formed by the atomic planes and the entrance surface (Fig. 1). The field of any frequency in the crystal is represented by a two-wave approximation as

$$\mathbf{E} = \mathbf{E}_0\exp(i\mathbf{K}_0\mathbf{r}) + \mathbf{E}_h\exp(i\mathbf{K}_h\mathbf{r}),$$

where $\mathbf{K}_0$ and $\mathbf{K}_h = \mathbf{K}_0 + \mathbf{h}$ vectors are the wavevectors of the transmitted and diffracted fields, respectively, which satisfy the Bragg exact condition for a certain wavelength, and $\mathbf{K}_0^2 = \mathbf{K}_h^2 = (2\pi/\lambda)^2$, $\mathbf{E}_0$, $\mathbf{E}_h$ are the amplitudes of the electric fields of the transmitted and diffracted waves for the same wavelength. The amplitudes satisfy Takagi's equations of dynamic diffraction (Takagi, 1969),

$$\begin{aligned} &2ik\big[\cos(\theta_0 - \alpha)\partial/\partial x + \sin(\theta_0 - \alpha)\partial/\partial z\big]E_0 + k^2\chi_0 E_0 \\ &\quad + k^2\chi_{\bar h} C E_h = 0, \\ &2ik\big[\cos(\theta_0 + \alpha)\partial/\partial x - \sin(\theta_0 + \alpha)\partial/\partial z\big]E_h + k^2\chi_0 E_h \\ &\quad + k^2\chi_h C E_0 = 0. \end{aligned} \tag{32}$$

Here C is the polarization factor; $C = 1$ for σ-polarization and $C = \cos 2\theta_0$ for π-polarization. Hereinafter the polarization factor is omitted. On the first-crystal entrance surface the E_0 field corresponding to any polarized state satisfies the continuity condition

$$E_0(x, y) = E^i(x, y)\exp(-ik\Delta\theta\gamma_0 x). \tag{33}$$

Here $\Delta\theta = \theta - \theta_0$ is the deviation of the wavevector $\mathbf{K}$ from the exact Bragg angle for a certain wavelength, and the linear approximation by that deviation is used. It is known that in the Bragg case the diffracted field amplitude at the crystal entrance surface can be expressed by the convolution of the point-source function and the incident wave amplitude (Uragami, 1969),

$$E_h(x, y, 0) = \int_{-\infty}^{+\infty} G_{h0}(x - x')E_0(x', y, 0)\,\mathrm{d}x', \tag{34}$$

where

$$G_{h0}(x) = i\left(\frac{\chi_h}{\chi_{\bar h}}\right)^{1/2}\left(\frac{\gamma_0}{\gamma_h}\right)^{1/2}\frac{J_1(\sigma x)}{x}\exp(i\sigma_0 x)H(x) \tag{35}$$

is the first-crystal point-source function, J_1 is the first-order Bessel function, $H(x)$ is the step function; $H(x) = 1$ when $x > 0$ and $H(x) = 0$ when $x < 0$. After reflection from the first plate the beam falls onto the lens. The lens is placed perpendicular to the diffracted beam. Since the case of a polychromatic beam is discussed, if it is assumed that the lens is placed perpendicular to $\mathbf{K}_h$, the direction of which depends on the wavelength, then clearly the perpendicularity condition of the lens to the diffracted beam will be broken for another wavelength, that is not suitable for calculations. Since the directions of the diffracted waves for different wavelengths together form angles of $\sim 20''$, it is assumed that the lens is placed perpendicular to the direction which forms angle $(\theta + \alpha)$ with the entrance surface of the first crystal. Such a choice does not depend on the wavelength and is defined only by the direction of the incident beam. So, both in the lens and in the gap the electric field corresponding to any polarization state and any frequency in the form $E_h'\exp(ik\mathbf{s}_h\mathbf{r})$ is represented, where $\mathbf{s}_h = [\cos(\theta + \alpha), -\sin(\theta + \alpha)]$ is a unit vector, the direction of which does not depend on the wavelength. The lens is placed perpendicular to the direction defined by vector $\mathbf{s}_h$. If $L_G \ll L_s$, where L_G is the characteristic size of the gap in the direction of $\mathbf{s}_h$, and L_s is the characteristic distance between the source and the system, then E_h' in the gap and in the lens propagates by $x + zc\tan(\theta + \alpha) = $ constant characteristics parallel to $\mathbf{s}_h$, acquiring an additional phase in the lens. In the gap and in the lens the field amplitude propagates according to the following equations, respectively,

$$\begin{aligned} &2ik\big[\cos(\theta + \alpha)\partial E_h'/\partial x - \sin(\theta + \alpha)\partial E_h'/\partial z\big] = 0, \\ &2ik\big[\cos(\theta + \alpha)\partial E_h'/\partial x - \sin(\theta + \alpha)\partial E_h'/\partial z\big] + k^2\chi_{0l}E_h' = 0, \end{aligned} \tag{36}$$

where χ_{0l} is the dielectric susceptibility of the lens and is related to the refractive index of the lens by the relation $n = 1 + \chi_{0l}/2$. The first equation of (36) can be obtained from the second equation of (32) by replacing $\chi_0 = \chi_h = 0$, and the second equation of (36) can be obtained from the second equation of (32) by replacing $\chi_0 = \chi_{0l}$, $\chi_h = 0$. In both cases θ must be taken instead of θ_0. The amplitude E_h' satisfies the continuity conditions on both surfaces of the lens and on the surface of the first plate. Using (36) the amplitude on the exit surface of the lens is

$$E_h'(x, y, z) = E_h'(\xi_h, y, 0)\exp\left(ik\frac{\chi_{0l}}{2}\frac{\xi_h^2}{R}\right)\exp\left(ik\frac{\chi_{0l}}{2}T_0\right), \tag{37}$$

where $\xi_h = x\sin(\theta + \alpha) + z\cos(\theta + \alpha)$ and T_0 is the lens thickness at the apex. The entrance and exit surfaces of the lens are given by $\eta_{h\,\mathrm{ent}} = L_h - (\xi_h^2/2R)$ and $\eta_{h\,\mathrm{ex}} = L_h + T_0 + (\xi_h^2/2R)$, respectively. Here $\eta_h = x\cos(\theta + \alpha) - z\sin(\theta + \alpha)$ and L_h is the distance of the apex of the lens from the origin O. Taking into account that after the lens the wave amplitude propogates according to the first equation of (36) and satisfies the continuity condition on the exit surface of the lens, the amplitude in the gap after the lens is represented as

$$
\begin{aligned}
E'_h(x, y, z) = {} & E_h[x + zc\tan(\theta+\alpha), y, 0] \\
& \times \exp\{ik\Delta\theta\gamma_h[x + zc\tan(\theta+\alpha)]\} \\
& \times \exp[ik(\chi_{01}/2)T_0] \\
& \times \exp\left\{ik\frac{\chi_{01}\sin^2(\theta+\alpha)}{2}\frac{[x+zc\tan(\theta+\alpha)]^2}{R}\right\}.
\end{aligned} \tag{38}
$$

Passing through the lens and the gap the wave falls on the second plate. In the second plate the field also satisfies Takagi's equations (32) and the continuity conditions $E'_h\exp(ik\mathbf{s}_h\mathbf{r}) = E_h\exp(i\mathbf{K}_h\mathbf{r})$ on the surface of the second plate ($z = -D$), where D is the size of the gap in the z direction. Once again applying the formula (34) but with the point-source function of the second crystal,

$$
G_{0h}(x) = i\left(\frac{\chi_{\bar h}}{\chi_h}\right)^{1/2}\left(\frac{\gamma_h}{\gamma_0}\right)^{1/2}\frac{J_1(\sigma x)}{x}\exp(i\sigma_0 x)H(x), \tag{39}
$$

the amplitude of the double diffracted beam on the surface of the second crystal is represented as

$$
E_0(x, y, -D) = \int G_{0h}(x - x')E_h(x', y, -D)\,\mathrm{d}x', \tag{40}
$$

where

$$
E_h = E'_h\exp(-ik\Delta\theta\gamma_h x)\exp\left[-i\left(ks_{hz} - K_{hz}\right)D\right]. \tag{41}
$$

Later the constant phase exponent in (41) can be omitted, since it has no effect on the final result. Inserting (41) and (38) into (40) for the double-diffracted wave amplitude on the surface of the second plate, the following equation is obtained,

$$
\begin{aligned}
E_0(x, y, -D) = {} & \exp\left(ik\frac{\chi_{01}}{2}T_0\right)\iint G_{0h}(x - x') \\
& \times G_{h0}[x' - Dc\tan(\theta+\alpha) - x'']E_0(x'', y, 0) \\
& \times \exp\left\{ik\frac{\chi_{01}\sin^2(\theta+\alpha)}{2}\frac{[x' - Dc\tan(\theta+\alpha)]^2}{R}\right\} \\
& \times \mathrm{d}x'\,\mathrm{d}x'' \\
= {} & \exp\left(ik\frac{\chi_{01}}{2}T_0\right)\iint G_{0h}[x - Dc\tan(\theta+\alpha) - x'] \\
& \times G_{h0}(x' - x'')\exp\left[ik\frac{\chi_{01}\sin^2(\theta+\alpha)}{2}\frac{x'^2}{R}\right] \\
& \times E_0(x'', y, 0)\,\mathrm{d}x'\,\mathrm{d}x''.
\end{aligned} \tag{42}
$$

The double-diffracted field W^e_{0h} propagating in the vacuum is represented in the form $W^e_{0h} = E^e_{0h}\exp(i\mathbf{K}\mathbf{r})$. It satisfies the Helmholtz equation

$$
\frac{\partial^2 W^e_{0h}}{\partial x^2} + \frac{\partial^2 W^e_{0h}}{\partial y^2} + \frac{\partial^2 W^e_{0h}}{\partial z^2} + k^2 W^e_{0h} = 0.
$$

From this equation we can find the propagation equation for E^e_{0h},

$$
\begin{aligned}
& \frac{\partial^2 E^e_{0h}}{\partial x^2} + \frac{\partial^2 E^e_{0h}}{\partial y^2} + \frac{\partial^2 E^e_{0h}}{\partial z^2} \\
& \quad + 2ik\left[\cos(\theta-\alpha)\frac{\partial}{\partial x} + \sin(\theta-\alpha)\frac{\partial}{\partial z}\right]E^e_{0h} = 0.
\end{aligned} \tag{43}
$$

Since the first derivatives in (43) are large in comparison with the second derivatives, in parabolic approximation from (43) one can write the shortened propagation equation,

$$
\begin{aligned}
& \frac{1}{\sin^2(\theta-\alpha)}\frac{\partial^2 E^e_{0h}}{\partial x^2} + \frac{\partial^2 E^e_{0h}}{\partial y^2} \\
& \quad + 2ik\left[\cos(\theta-\alpha)\frac{\partial}{\partial x} + \sin(\theta-\alpha)\frac{\partial}{\partial z}\right]E^e_{0h} = 0.
\end{aligned} \tag{44}
$$

Equation (44) can be obtained by making the Fourier transformation of (43), writing the obtained dispersion equation in parabolic approximation. After making the inverse Fourier transformation of this approximated dispersion equation, (44) is obtained. The wave amplitude in a vacuum can be represented by means of the Green function corresponding to that equation. According to the definition, the Green function should satisfy the differential equation

$$
\begin{aligned}
& \frac{1}{\sin^2(\theta-\alpha)}\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} - 2ik\left[\cos(\theta-\alpha)\frac{\partial}{\partial x} + \sin(\theta-\alpha)\frac{\partial}{\partial z}\right]G \\
& \quad = 2ik\sin(\theta-\alpha)\delta(\mathbf{r} - \mathbf{r}_p),
\end{aligned} \tag{45}
$$

where $\delta(\mathbf{r} - \mathbf{r}_p)$ is the Dirac three-dimensional δ-function. The coefficient $-2ik\sin(\theta-\alpha)$ is introduced for convenience. Taking a volume Q_v restricted with the surface Q, multiplying (44) by G, then adding (45) multiplied by $-E^e_{0h}$ and integrating the result in the volume Q_v using the Gauss theorem, one can obtain the amplitude E^e_{0h} at the point $\mathbf{r}_p$ of the volume Q_v represented by means of the Green function,

$$
\begin{aligned}
& \frac{1}{\sin^2(\theta-\alpha)}\oint\left(G\frac{\partial E^e_{0h}}{\partial x} - E^e_{0h}\frac{\partial G}{\partial x}\right)\mathrm{d}Q_x \\
& + \oint\left(G\frac{\partial E^e_{0h}}{\partial y} - E^e_{0h}\frac{\partial G}{\partial y}\right)\mathrm{d}Q_y + 2ik\cos(\theta-\alpha) \\
& \times \oint GE^e_{0h}\,\mathrm{d}Q_x + 2ik\sin(\theta-\alpha)\oint GE^e_{0h}\,\mathrm{d}Q_z \\
& = -2ik\sin(\theta-\alpha)E^e_{0h}(\mathbf{r}_p).
\end{aligned} \tag{46}
$$

For representation of the amplitude in the form (46) it is necessary to find the Green function. Solving (45) by Fourier methods the following is obtained,

$$
\begin{aligned}
& G\left(x_p - x, y_p - y, z_p - z\right) = \\
& \quad -\frac{ik}{2\pi}\frac{\sin^2(\theta-\alpha)}{(z_p - z)}\exp\left[ik\frac{\left(\xi_{0p} - \xi_0\right)^2\sin(\theta-\alpha)}{2(z_p - z)}\right] \\
& \quad \times\exp\left[ik\frac{\left(y - y_p\right)^2\sin(\theta-\alpha)}{2\left(z_p - z\right)}\right]H\left(z_p - z\right),
\end{aligned} \tag{47}
$$

where $\xi_0 = [x - zc\tan(\theta-\alpha)]\sin(\theta-\alpha)$ and $\xi_{0p} = [x_p - z_p c\tan(\theta-\alpha)]\sin(\theta-\alpha)$. Now the amplitude of the double-diffracted wave can be represented as an integral over the surface of the second plate if the surface of the second plate is taken as part of the integration surface in (46). On the infinite surface, which closes the volume Q_v with the surface of the second plate, the integral for the physical waves tends to zero. For the surface of the second plate, all components $\mathrm{d}Q_i$ are zero, instead of $\mathrm{d}Q_z = -\mathrm{d}x\,\mathrm{d}y$. Therefore, from (46),

$$E^e_{0h}(\mathbf{r}_p) = \int G(x_p - x, y_p - y, z_p + D) E^e_{0h}(x, y, -D)\, \mathrm{d}x\, \mathrm{d}y. \tag{48}$$

$E^e_{0h}(x, y, -D)$ can be found from the continuity conditions on the surface of the second plate,

$$E^e_{0h}(x, y, -D) \exp(i\mathbf{K}\mathbf{r}) = E_0(x, y, -D) \exp(i\mathbf{K}_0\mathbf{r}).$$

From this condition,

$$\begin{aligned} E^e_{0h}(x, y, -D) &= E_0(x, y, -D) \exp\big[i(\mathbf{K}_0 - \mathbf{K})\mathbf{r}\big] \\ &= E_0(x, y, -D) \\ &\quad \times \exp\big\{ik\big[\cos(\theta_0 - \alpha) - \cos(\theta - \alpha)\big]x\big\} \\ &\quad \times \exp\big\{-ik\big[\sin(\theta_0 - \alpha) - \sin(\theta - \alpha)\big]D\big\}. \end{aligned}$$

Again omitting the non-essential constant phase exponents $E^e_{0h}(x, y, -D) = E_0(x, y, -D) \exp(ik\Delta\theta\gamma_0 x)$, then inserting this expression into (48), the following can be written,

$$\begin{aligned} E^e_{0h}(\mathbf{r}_p) = {} & \int G\big(x_p - x, y_p - y, z_p + D\big) E_0(x, y, -D) \\ & \times \exp(ik\Delta\theta\gamma_0 x)\, \mathrm{d}x\, \mathrm{d}y. \end{aligned} \tag{49}$$

Now using (42) and (49) the amplitude of the double-diffracted beam can be represented as

$$\begin{aligned} E^e_{0h}(\mathbf{r}_p) = {} & \exp\Big(ik\frac{\chi_{01}}{2}T_0\Big) \int G\big(x_p - x, y_p - y, z_p + D\big) \\ & \times G_{0h}[x - Dc\tan(\theta + \alpha) - x'] G_{h0}(x' - x'') \\ & \times \exp\left[ik\frac{\chi_{01}\sin^2(\theta + \alpha)}{2}\frac{x'^2}{R}\right] E_0(x'', y, 0) \\ & \times \exp(ik\Delta\theta\gamma_0 x)\, \mathrm{d}x\, \mathrm{d}x'\, \mathrm{d}x''\, \mathrm{d}y. \end{aligned} \tag{50}$$

Making the following series of transformation of variables in integral (50), $x' - x'' \to x''$, $x - Dc\tan(\theta + \alpha) - x' \to x'$, $x - Dc\tan(\theta + \alpha) - x' \to x$,

$$\begin{aligned} E^e_{0h}(\mathbf{r}_p) = {} & \exp\Big(ik\frac{\chi_{01}}{2}T_0\Big) \int \mathrm{d}x' \int\limits_{-R_{ox-x'}}^{R_{ox-x'}} \mathrm{d}x \\ & \times \int G\big[x_p - Dc\tan(\theta + \alpha) - x - x', y_p - y, z_p + D\big] \\ & \times G_{0h}(x') G_{h0}(x'') \exp\left[ik\frac{\chi_{01}\sin^2(\theta + \alpha)}{2}\frac{x^2}{R}\right] \\ & \times E_0(x - x'', y, 0) \exp\big[ik\Delta\theta\gamma_0(x + x')\big]\, \mathrm{d}x''\, \mathrm{d}y. \end{aligned}$$

Here the non-essential constant phase exponents are omitted. Taking into account that $R_{0x} \gg 1/\sigma_{\rm r}$ [$1/\sigma_{\rm r}$ is the half width of $G_{0h}(x')$], the limits of integration $(-R_{0x} - x', R_{0x} + x')$ by x can be taken independent of x' as $(-R_{0x}, R_{0x})$. Then, the amplitude of the double-diffracted beam can be represented as

$$\begin{aligned} E^e_{0h}(\mathbf{r}_p) = {} & \exp\Big(ik\frac{\chi_{01}}{2}T_0\Big) \int\limits_{-Rox}^{Rox} \mathrm{d}x \\ & \times \int G\big[x_p - Dc\tan(\theta + \alpha) - x - x', y_p - y, z_p + D\big] \\ & \times G_{0h}(x') G_{h0}(x'') \exp\left[ik\frac{\chi_{01}\sin^2(\theta + \alpha)}{2}\frac{x^2}{R}\right] \\ & \times E_0(x - x'', y, 0) \exp[ik\Delta\theta\gamma_0(x + x')]\, \mathrm{d}x'\, \mathrm{d}x''\, \mathrm{d}y. \end{aligned} \tag{51}$$

Here the limits of integration by x are shown; the inner integrals are taken in the limits $(-\infty, \infty)$. Denoting $x_p \equiv x_p - Dc\tan(\theta + \alpha)$, $z_p \equiv z_p + D$, $\xi_p = [x_p - z_p c\tan(\theta - \alpha)]\sin(\theta - \alpha)$, $\xi = x\sin(\theta - \alpha)$, $\xi' = x'\sin(\theta - \alpha)$, $L = z_p / \sin(\theta - \alpha)$, from (51) one can obtain

$$\begin{aligned} E^e_{0h}(\mathbf{r}_p) = {} & \exp\Big(ik\frac{\chi_{01}}{2}T_0\Big) \\ & \times \int\limits_{-Rox}^{Rox} \int G\big(x_p - x - x', y_p - y, z_p\big) \\ & \times G_{0h}(x') G_{h0}(x'') \exp\left[ik\frac{\chi_{01}\sin^2(\theta + \alpha)}{2}\frac{x^2}{R}\right] \\ & \times E_0(x - x'', y, 0) \exp\big[ik\Delta\theta\gamma_0(x + x')\big]\, \mathrm{d}x\, \mathrm{d}x'\, \mathrm{d}x''\, \mathrm{d}y \end{aligned} \tag{52}$$

where

$$G = -\frac{ik\sin(\theta - \alpha)}{2\pi L} \exp\left[ik\frac{(\xi_p - \xi' - \xi)^2}{2L}\right] \exp\left[ik\frac{(y_p - y)^2}{2L}\right]. \tag{53}$$

According to the obtained formula, the amplitude is a function of variables ξ_p and L, which are calculated from the point O' (see Fig. 1). σ-polarized waves are considered, and for π-polarization χ_h and $\chi_{\bar{h}}$ should be multiplied by $\cos 2\theta_0$. Hereafter, for the integrals taken in the limits $(-\infty, +\infty)$, the limits are not indicated.

Formula (52) is the main formula for the amplitude of the double-diffracted beam for an arbitrary incident wave. Taking the incident wave amplitude for various cases and inserting into (52), the amplitude for the double-diffracted beam can be obtained. In this paper this procedure will be carried out for incident plane and spherical waves. The second case corresponds to imaging of a point source by the RDL. The focus point properties are considered by means of (52). The results for focusing distance and magnification are also obtained in a wave optical consideration.

4. Plane-wave focusing

One of the cases of interest from a physical aspect is the case when the distance between the X-ray source and the RDL is very large. Then in the phase of the incident wave the quadratic term can be neglected. The plane-wave approximation condition is given by (24). Leaving only the linear term, the plane-wave approximation is used. Taking $E^i(x, y)$ = constant = E^i in the boundary condition (33) and inserting it into (52), then integrating by y, the amplitude has the following expression,

$$E^e_{0h}(\mathbf{r}_p) = E^i \exp\left(ik\frac{\chi_{01}}{2}T_0\right) \times \int_{-Rox}^{Rox} \mathrm{d}x \int G_x(x_p - x - x', L_0)G_{0h}(x')G_{h0}(x'') \times \exp\left[ik\frac{\chi_{01}\sin^2(\theta+\alpha)}{2}\frac{x^2}{R}\right] \times \exp\left[ik\Delta\theta\gamma_0(x'+x'')\right]\mathrm{d}x'\,\mathrm{d}x'', \quad (54)$$

where

$$G_x(x_p - x - x', L_0) = \gamma_0 \exp(-i\pi/4)(k/2\pi L)^{1/2} \times \exp\left[ik(\xi_p - \xi - \xi')^2/2L\right]. \quad (55)$$

In this case the reflection from the first crystal forms a plane wave. The integration describing this phenomenon is made by x''. Using the well known Fourier representation of the plane-wave solution by the point-source function (Gabrielyan *et al.*, 1989), the following formula is obtained,

$$\int G_{h0}(x'')\exp(ik\Delta\theta\gamma_0 x'')\,\mathrm{d}x'' = -\left(\frac{\chi_h}{\chi_{\bar{h}}}\right)^{1/2}\left(\frac{\gamma_0}{\gamma_h}\right)^{1/2}\frac{\sigma}{k\Delta\theta\gamma_0 + \sigma_0 + \left[(k\Delta\theta\gamma_0+\sigma_0)^2 - \sigma^2\right]^{1/2}}. \quad (56)$$

The next step is to integrate by x. Equation (55) is inserted into (54) and, instead of integrating by x, integration is made by the variable $\xi = x\sin(\theta-\alpha)$,

$$\int_{-R_{0x}\gamma_0}^{R_{0x}\gamma_0} \exp\left\{ik\frac{\xi^2}{2}\left[\frac{1}{L} + \chi_{01}\frac{\sin^2(\theta+\alpha)}{\sin^2(\theta-\alpha)}\frac{1}{R}\right] - ik\frac{(\xi_p - \xi')}{L}\xi\right\}\mathrm{d}\xi. \quad (57)$$

It is appropriate to discuss the cases of absorptive and non-absorptive lenses separately.

4.1. Non-absorptive lens

For a non-absorptive lens, χ_{01} is real. It is clear that when the limits of integration are taken $(-\infty, +\infty)$, the integral value of (57) is infinite when

$$L = F = F_0 b^2, \qquad \xi_p = \xi', \quad (58)$$

where $b = \sin(\theta-\alpha)/\sin(\theta+\alpha)$ and $F_0 = -R/\chi_{01}$ is the focal distance of the separately taken lens. So, the RDL focal distance is determined by the formula (58), which is obtained in §2 in the geometric optical sense [formula (15)]. At the focal distance defined by (58) the integral (57) gives $2\sin[k(\xi_p-\xi')R_{0x}\gamma_0/F]/[k(\xi_p-\xi')/F]$. If its half-width is smaller than the first zero of $G_{0h}(\xi'/\gamma_0)$, *i.e.* $3.8(\gamma_0/\sigma_r)/[2F/(k\gamma_0R_{0x})] > 1$, where σ_r is the real part of σ, it can be replaced by the δ-function $2\pi F\delta(\xi_p-\xi')/k$. This is the case for a large lens. For the Si(220) (Mo $K\alpha$) reflection and for a lens made of beryllium ($\delta = 1.118\times10^{-6}$, see below) with R = 1 mm, R_0 = 1 mm, R_{0x} = 3 mm, it follows that $3.8(\gamma_0/\sigma_r)/[2F/(k\gamma_0R_{0x})] \simeq 3.7$. This approximation will be more correct if R_0 is greater. For example, when R_0 = 2 mm, then $3.8(\gamma_0/\sigma_r)/[2F/(k\gamma_0R_{0x})] \simeq 7.53$. Taking $R_{0x} > 2F\sigma_r/3.8k\gamma_0^2$ and replacing $2\sin[k(\xi_p-\xi')R_{0x}\gamma_0/F]/[k(\xi_p-\xi')/F]$ by $2\pi F\delta(\xi_p-\xi')/k$, and inserting it, as well as (55) and (56), into (54), the amplitude value on the focal plane is obtained,

$$E^e_{0h}(\mathbf{r}_p) = -E^i\exp(i\pi/4)\exp\left(ik\frac{\chi_{01}}{2}T_0\right)\Gamma(\Delta\theta) \times \left(\frac{2\pi F}{k}\right)^{1/2}\frac{J_1(\sigma\xi_p/\gamma_0)}{\xi_p}\exp(i\sigma_0\xi_p/\gamma_0) \times \exp(ik\Delta\theta\xi_p)H(\xi_p), \quad (59)$$

where

$$\Gamma(\Delta\theta) = \frac{\sigma}{k\Delta\theta\gamma_0 + \sigma_0 + \left[(k\Delta\theta\gamma_0+\sigma_0)^2 - \sigma^2\right]^{1/2}}.$$

Note that in (59), and later on, wherever the difference of γ_0 and $\sin(\theta-\alpha)$, as well as γ_h and $\sin(\theta+\alpha)$, is insignificant, γ_0 is written instead of $\sin(\theta-\alpha)$ and γ_h instead of $\sin(\theta+\alpha)$.

From (59) it follows that the RDL plane-wave focusing in the focal plane for the amplitude E^e_{0h} gives the point-source function of the Bragg case. From this point of view, like in optics [theory of Abbe (Born & Wolf, 1968)], in the microscope case, the second-crystal surface wavefield can be considered as an object imaged through a lens on the focal plane (Fourier transform of the Bragg-reflected incident plane wave). From the expression defined by (59) one can see that the field is focused at the point $\xi_{pf} = 0$. Therefore, summarizing the obtained results, the focal distance and coordinate of the focus point on the focal plain are

$$F = F_0 b^2, \qquad \xi_{pf} = 0. \quad (60)$$

Both the focal distance and the focus coordinate do not depend on the wavelength. The components corresponding to all the wavelengths of the polychromatic plane wave are focused at the same point. However, their intensities depend on the deviation from the Bragg condition corresponding to each wavelength.

Formula (59) allows the focus size on the focal plane to be evaluated. It is determined from the condition $\sigma_r\Delta\xi_{pf}/\gamma_0 \simeq 1$ as

$$\Delta\xi_{pf} \simeq \gamma_0/\sigma_r. \quad (61)$$

Here σ_r is the real part of σ. Taking $\gamma_0 = 0.017$, $\gamma_h = 0.36$ from (61) for Si(220) (Mo $K\alpha$) radiation, $\Delta\xi_{pf} \simeq 0.5$ µm is estimated.

Then the value of the intensity at the focus for a certain wavelength is

$$I_f(0, F) = \left|E^e_{0f}(0, F)\right|^2 = \left|E^i\right|^2\left|\Gamma(\Delta\theta)\right|^2\frac{2\pi F}{k}\left|\frac{\sigma}{2\gamma_0}\right|^2. \quad (62)$$

Taking the same parameters and a lens made of beryllium, $I_f(0, F) = 84|E^i|^2$ is estimated from (62).

4.1.1. Angular resolution. Note that plane waves falling at different angles θ are focused at different distances, and on the focal plane the focus lies in the central direction, which makes an angle $(\theta - \alpha)$ with the second-crystal surface. Since plane waves which form angles in the small-angle range are considered, the focal distance difference can be neglected, and in the general focal plane the absolute value of the distance between two arbitrary plane-wave foci calculated in the direction perpendicular to the double-reflected beam in the diffraction plane is $|\xi_{pf1} - \xi_{pf2}| = F|\theta_1 - \theta_2|$, while the reversed image appears for the corresponding plane-wave sources. There is a possibility of using RDLs in X-ray astronomy. From (59) the angular resolution of a RDL can be estimated. From (59), according to the Rayleigh criterion, two plane waves are angularly resolved when $F|\theta_1 - \theta_2| \simeq 3.8\gamma_0/\sigma_r$. Here, 3.8 is the first zero of the point-source function in (59). Therefore for the resolved angle the following estimation is true,

$$|\theta_1 - \theta_2| \simeq 3.8\gamma_0/(\sigma_r F). \tag{63}$$

For the case considered in §2 [Si220 (Mo $K\alpha$) radiation, lens made of beryllium], one can estimate $|\theta_1 - \theta_2| \simeq 1.83 \times 10^{-6} = 0.37''$. Now it is interesting to compare the resolution of the RDL with the resolution of the individual cylindrical lens with the same parameters as for the lens placed in the gap. The intensity distribution of a cylindrical lens in the focal plane is given by the Fraunhofer diffraction formula on a slit with the same aperture as of the lens, $I_f(x) = 4R_0^2(k/2\pi F_0)[\sin(kxR_0/F_0)/(kxR_0/F_0)]^2$, where x is a coordinate in the focal plane. The first zero of this distribution is at $kxR_0/F_0 = \pi$. The two plane waves are resolved if $|\theta_1 - \theta_2| \simeq \pi/(kR_0)$. For Mo $K\alpha$ radiation, $k \simeq 10^{11}$ m^{-1}, and if R_0 = 1 mm then $|\theta_1 - \theta_2| \simeq 10^{-8}\pi = 0.006''$, *i.e.* it is two orders smaller than that for the RDL. However, the advantage of the RDL is the small focal distance; the focal distance of the RDL is smaller by a factor of 400 than that of the individual lens, which makes the RDL appropriate for applications ($F \simeq 1$ m). The focal distance of the individual lens is so large, $F_0 \simeq$ 500 m, that its application encounters principle difficulties.

4.1.2. Fourier method. For investigation of the intensity distribution on the focusing line of RDLs it is more convenient to refer to the expression of the Fourier representation of the amplitude, because in the focal plane the δ-function appears. Using table integrals, the convolution Fourier analysis theorem and (56),

$$E^e_{0h}(\mathbf{r}_p) = -E^i \exp(-i\pi/4)\exp\left(ik\frac{\chi_{0\mathrm{i}}}{2}T_0\right)\left(\frac{F}{2\pi k\gamma_0^2}\right)^{1/2}\sigma\Gamma(\Delta\theta) \times \int \frac{\exp\left[-iq^2(L-F)/2k + iq\xi_p\right]}{q' - \left(q'^2 - \sigma^2/\gamma_0^2\right)^{1/2}}\,\mathrm{d}q, \tag{64}$$

where $q' = q - (\sigma_0/\gamma_0) - k\Delta\theta$. It is easy to note that (59) is obtained from (64) in the focal plane.

By using (64) and the integral calculation stationary phase method, the directions of the trajectories can be determined. The sub-integral exponent phase is designated by $\Phi(q) = q\xi_p - q^2(L-F)/2k$. According to the stationary phase method the stationary points are defined by the condition $\mathrm{d}\Phi(q)/\mathrm{d}q = 0$, which in our case gives $\xi_p - (L-F)q/k = 0$. It follows that the trajectories determined by different q values represent straight lines, which at the distance $L = F$ pass through the point $\xi_p = 0$ independently of q. The point $(0, F)$ is where all the trajectories intersect, *i.e.* the focus, after which the trajectories diverge. The coordinates of this point are independent of the wavelength, and again all the wavelengths are gathered in one point. This stationary phase method is equivalent to the geometric optical method, described in §2 for an incident plane wave.

4.1.3. Example. Let us discuss a particular case, the Si(220) Mo $K\alpha_1$ reflection (λ_m = 0.709 Å). The non-absorptive lens is made of beryllium. The index of refraction of the lens is defined as $n = 1 + \chi_{0l}/2 = 1 + \chi_{0lr}/2 + i\chi_{0li}/2 = 1 - \delta - i\beta$. Here, χ_{0lr} and χ_{0li} are the real and imaginary parts of the lens susceptibility. It is assumed that R = 1 mm and the lens aperture is 2 mm. In this case the largest thickness of the lens is $T \simeq 1$ mm if T_0 = 0.1 mm. The intensity absorption in the lens takes place according to the $\exp(-\mu T)$ rule, where $\mu = 4\pi|\beta|/\lambda$ is the linear absorption coefficient of the lens. Taking the absorption coefficient by mass μ/ρ = 0.0257 m^2 kg^{-1} and the beryllium density ρ = 1850 kg m^{-3} from the http://lipro.msl.titech.ac.jp/eindex.html (Materials and Structures Laboratory, University of Tokyo) website, the linear absorption coefficient μ = 47.6 m^{-1} for Mo $K\alpha_1$ radiation is calculated. Now we can estimate $\mu T \simeq 0.05$. Therefore, the absorption in the lens can be neglected. Using the relation between μ and β, $\beta = -2.69 \times 10^{-10}$ is obtained. δ is calculated using the well known formula (Lengeler *et al.*, 1999). According to the above-mentioned website, the dispersion correction with the third digit accuracy f' = 0. Using these data, $\delta = 1.118 \times 10^{-6}$. For the Si(220) Mo $K\alpha_1$ asymmetric reflection, $\theta_0 = 10.626°$. If $\alpha = 9.626°$, then $\gamma_0 = 0.0175$, γ_h = 0.346, b = 0.05, F = 1.14 m, $F_0 = R/2\delta$ = 448.43 m.

In the spectrum of the polychromatic beam there is a wavelength satisfying the Bragg-corrected condition, *i.e.* $k\Delta\theta\gamma_0 + \sigma_{0r} = 0$ for this wave [see the expression for $\Gamma(\Delta\theta)$]. It is natural to assume that the beam falls at the Bragg-corrected angle for the wave component with maximum intensity. In this case, $k\Delta\theta_m\gamma_0 + \sigma_{0r} = 0$, where $\Delta\theta_m = \theta - \theta_0(\lambda_m)$, and $\Delta\theta = \theta - \theta_0(\lambda) = \theta - \theta_0(\lambda_m) + \theta_0(\lambda_m) - \theta_0(\lambda) = \Delta\theta_m - (\Delta\lambda/\lambda_m)\tan\theta$. Using this relation between $\Delta\theta$ and $\Delta\lambda$, one can refer to the wavelength dependence in the $\Gamma(\Delta\theta)$ function,

$$\Gamma(\Delta\lambda) = \frac{\sigma}{\sigma_1(\Delta\lambda) + \left[\sigma_1(\Delta\lambda)^2 - \sigma^2\right]^{1/2}},$$

where $\sigma_1(\Delta\lambda) = i\sigma_{0i} - k\gamma_0\tan\theta(\Delta\lambda/\lambda_m)$. From this expression it follows that these wavelengths reflect effectively, and $|\Delta\lambda/\lambda_m| \leq \sigma_r c \tan\theta/k\gamma_0$. For the case under consideration this range is $\sim 10^{-4}$. If the wavelength $\lambda = \lambda_m$ is fixed and the beam angular deviation is calculated as $\Delta\theta_c = \theta - \theta_c(\lambda_m)$, where $\theta_c(\lambda_m)$ is the Bragg-corrected angle determined from the condition $k[\theta_c(\lambda_m) - \theta_0(\lambda_m)]\gamma_0 + \sigma_{0r} = 0$, then, taking into account $\Delta\theta = \Delta\theta_c + \theta_c(\lambda_m) - \theta_0(\lambda_m)$, one can refer to the $\Delta\theta_c$ dependence in the $\Gamma(\Delta\theta)$ function,

$$\Gamma(\Delta\theta_c) = \frac{\sigma}{\sigma_1(\Delta\theta_c) + \left[\sigma_1(\Delta\theta_c)^2 - \sigma^2\right]^{1/2}},$$

where $\sigma_1(\Delta\theta_c) = k\Delta\theta_c\gamma_0 + i\sigma_{0i}$. The $|\Gamma(\Delta\lambda)|^2$ and $|\Gamma(\Delta\theta_c)|^2$ dependences have the same behavior. This is why the intensity distributions given by (59) on the focal plane are presented only for different $\Delta\lambda/\lambda_m$ (Fig. 4).

The curve of the intensity dependence of L on the $\xi \to +0$ focal line can be obtained by means of the numerical integration of formula (64) (Fig. 5).

4.2. Absorptive lens

In the case of an absorptive lens, χ_{0l} is a complex number. Instead of the integral (57), the table integral can be written as

$$\int_{-R_{0x}\gamma_0}^{R_{0x}\gamma_0} \exp\left\{-k\left[\frac{|\beta|}{F\delta} + i\left(\frac{1}{F} - \frac{1}{L}\right)\right]\xi^2/2 - ik(\xi_p - \xi')\xi/L\right\} d\xi.$$

The case of a large lens is considered, where the limits can be taken as $(-\infty, \infty)$. In this case $k(|\beta|/F\delta)(R_{0x}\gamma_0)^2/2 \geq 1$. This case is realised for a lens made of silicon with R = 1 mm, R_0 = 1.5 mm, R_{0x} = 4.3 mm. Then $k(|\beta|/F\delta)(R_{0x}\gamma_0)^2/2$ = 1.65. For the case R_0 = 1 mm, one can find $k(|\beta|/F\delta)(R_{0x}\gamma_0)^2/2 \simeq 0.8$. For a lens made of silicon, F = 0.8 m, $F_0 = R/2\delta$ = 316.26 m and F_0/F = 400. The silicon data $\chi_{0r} = -3.162 \times 10^{-6}$, $\chi_{0i} = 0.165 \times 10^{-7}$, $\chi_{hr} = \chi_{\bar{h}r} = -1.901 \times 10^{-6}$, $\chi_{hi} = \chi_{\bar{h}i} = 0.159 \times 10^{-7}$ are derived from Pinsker (1982).

After taking the limits of integration $(-\infty, +\infty)$ and integrating, the amplitude has the following expression,

$$E^e_{0h}(\xi, L) = -E^i \exp(i\pi/4)\exp\left(ik\frac{\chi_{0l}}{2}T_0\right)\Gamma(\Delta\theta)(F/L)^{1/2} \times \left[\frac{\delta}{|\beta|(1+i\gamma)}\right]^{1/2} \int \exp(\Psi)\frac{J_1(\sigma\xi'/\gamma_0)}{\xi'}H(\xi')\,d\xi', \quad (65)$$

where

$$\gamma = \frac{\delta}{|\beta|}\left(1 - \frac{F}{L}\right),$$

$$\Psi = ik\frac{(\xi_p - \xi')^2}{2L} - s(\xi_p - \xi')^2 + \frac{i\sigma_0\xi'}{\gamma_0} + ik\Delta\theta\xi',$$

$$s = \frac{kF}{2L^2\left\{\left(|\beta|/\delta\right) + i[1 - (F/L)]\right\}}.$$

Since $|\beta|$ is much smaller than δ for X-ray radiation, all the essential conclusions made for the non-absorptive lens are true for this case.

For a lens made of silicon, Fig. 6 shows the intensity distribution using formula (65) on the focal plane for three different wavelengths ($R_0 \geq 1$ mm). The wavelengths are the same as in the non-absorptive case (see Fig. 4). The L curve of the intensity dependence on the focal line for the case $\lambda = \lambda_m$, $\theta = \theta_c(\lambda_m)$, is presented in Fig. 7 [numerical integration using formula (65)].

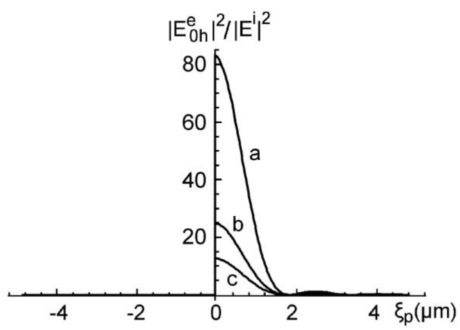

Figure 4
The double-diffracted beam intensity distribution on the focal plane in the RDL with a beryllium lens. Shown are the cases corresponding to $\Delta\lambda/\lambda_m = 0$ (*a*), $\Delta\lambda/\lambda_m = 1.5 \times 10^{-4}$ (*b*), $\Delta\lambda/\lambda_m = -1.9 \times 10^{-4}$ (*c*). The beam falls at the Bragg-corrected angle corresponding to the wave with $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is neglected. Incident plane wave.

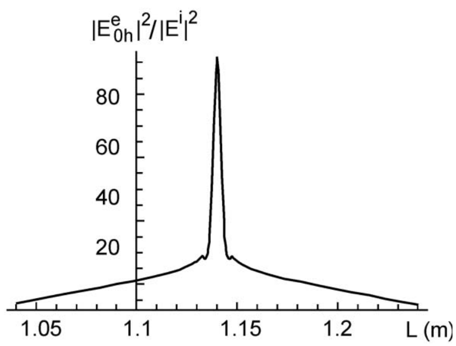

Figure 5
The intensity distribution on the $\xi \to +0$ focal line in a RDL with a beryllium lens. Incident plane monochromatic wave, $\Delta\lambda/\lambda_m = 0$, $\Delta\theta_c = 0$. Numerical integration is performed using formula (64). Absorption in the lens is neglected.

5. Formation of a point-source image

A point source emits a spherical wave

$$E = \exp(ikR_s)/R_s,$$

where R_s is the distance between the point source and the observation point. In the parabolic approximation on the entrance surface of the first plate,

$$E(x, y, \omega) = E^i(x, y, \omega)\exp(i\mathbf{Kr}) \quad (66)$$

with amplitude

$$E^i(x, y, \omega) = \frac{\exp(ikL_s)}{L_s}\exp\left[ikx^2\sin^2(\theta - \alpha)/2L_s\right] \times \exp\left(iky^2/2L_s\right). \quad (67)$$

Here L_s is the distance between the point source and the origin O, and x is also calculated from O. From the continuity condition on the entrance surface of the first plate,

$$E_0(x, y, \omega) = E^i(x, y, \omega)\exp(-ik\Delta\theta\gamma_0 x).$$

Inserting it into (52),

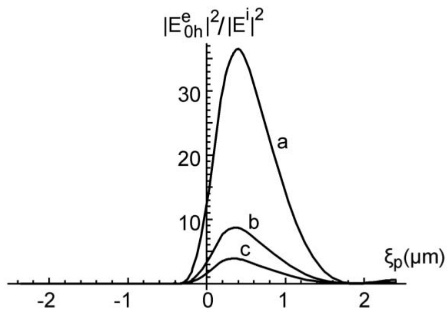

Figure 6
The intensity distribution of the focused beam on the focal plane in a RDL with a silicon lens. Shown are the cases corresponding to $\Delta\lambda/\lambda_m = 0$ (*a*), $\Delta\lambda/\lambda_m = 1.5 \times 10^{-4}$ (*b*), $\Delta\lambda/\lambda_m = -1.9 \times 10^{-4}$ (*c*). The beam falls at the Bragg-corrected angle corresponding to $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is taken into account. Computer calculation performed using formula (65). Incident plane wave.

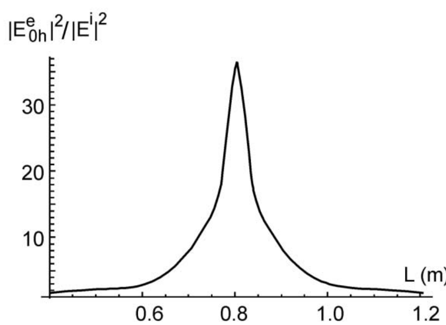

Figure 7
The intensity distribution on the focal line in a RDL with silicon lens. Plane monochromatic wave, $\Delta\lambda/\lambda_m = 0$, $\Delta\theta_c = 0$. Absorption in the lens is taken into account. Computer calculation performed using formula (65).

$$E^e_{0h}(\mathbf{r}) = \exp\left(ik\frac{\chi_{0l}}{2}T_0\right) \times \int G(x-x'-x'', y-y', z)G_{0h}(x'')G_{h0}(x''') \times \exp\left[ik\frac{\chi_{0l}\sin^2(\theta+\alpha)}{2}\frac{x'^2}{R}\right]E^i(x'-x''', y') \times \exp\left[ik\Delta\theta\gamma_0(x''+x''')\right]\,dx'\,dx''\,dx'''\,dy'. \qquad (68)$$

Here $\mathbf{r}$ is used instead of $\mathbf{r}_p$. Using the well known table integral the integration over y' can be made, after which the expression of the amplitude takes the form

$$E^e_{0h}(\mathbf{r}) = A_0\int G_\xi(\xi-\xi'-\xi'', y, z)G_{0h}(\xi''/\gamma_0)G_{h0}(\xi'''/\gamma_0) \times \exp\left(ik\frac{\chi_{0l}}{2}\frac{\xi'^2}{Rb^2}\right)E^i(\xi'-\xi''') \times \exp[ik\Delta\theta(\xi''+\xi''')]\,d\xi'\,d\xi''\,d\xi''', \qquad (69)$$

where

$$A_0 = \exp(-i\pi/4)\exp(ik\chi_{0l}T_0/2)\exp(ikL_s)/L_s\gamma_0^2,$$

$$G_\xi(\xi-\xi'-\xi'', y, z) = \exp\left[iky^2/2(L+L_s)\right] \times \left[kL_s/2\pi L(L+L_s)\right]^{1/2} \times \exp\left[ik(\xi-\xi'-\xi'')^2/2L\right],$$

$$E^i(\xi'-\xi''') = \exp\left[ik(\xi'-\xi''')^2/2L_s\right].$$

The non-absorptive and absorptive cases of the refractive lens must be considered separately.

5.1. Non-absorptive lens

In the case of a non-absorptive lens, χ_{0l} is real. In this case the integral over ξ' in (69) takes the form

$$\int \exp(ik\Phi_0)\,d\xi'/\sin(\theta-\alpha), \qquad (70)$$

where $\Phi_0 = A_1\xi'^2 + B_1\xi'$ and $A_1 = (1/L + 1/L_s + \chi_{0l}/Rb^2)/2$, $B_1 = [(\xi''-\xi)/L - \xi'''/L_s]$, $b^2 = \sin^2(\theta-\alpha)/\sin^2(\theta+\alpha)$. For large lenses, integral (70) gives $2\pi\delta[(\xi''-\xi)/L_f - \xi'''/L_s]/k$ at the focusing distance $L = L_f$ defined from the expression

$$1/L_s + 1/L_f = 1/F, \qquad (71)$$

where $F = -Rb^2/\chi_{0l} = F_0b^2$ is the focal distance of the RDL and $F_0 = -R/\chi_{0l}$ is the focal distance of the individual lens. The distance L_f is the distance where the image of the point source is formed. Using the obtained δ-function and integrating for the distance L_f in (69) over the variable ξ'' for the amplitude of the double-diffracted beam, one obtains

$$E^e_{0h} = A_0(2\pi F/k)^{1/2}\exp(ik\Phi_1)\int\exp(ik\Psi_1) \times G_{0h}\left[(\xi+\xi'''L_f/L_s)/\gamma_0\right]G_{h0}(\xi'''/\gamma_0)\,d\xi'''. \qquad (72)$$

Here $\Phi_1 = \xi\Delta\theta + y^2/2(L_f + L_s)$, $\Psi_1 = (1/2F)(L_f/L_s)\xi'''^2 + \xi'''\Delta\theta(1 + L_f/L_s)$.

Using the δ-function obtained from (70), the integration by ξ''' in (69) can also be made and the expression of the amplitude as an integral over the variable ξ'' is obtained,

$$E^e_{0h} = A_0(L_s/L_f)(2\pi F/k)^{1/2}\exp(ik\Phi_2)\int\exp(ik\Psi_2)G_{0h}(\xi''/\gamma_0) \times G_{h0}\left[(L_s/L_f)(\xi''-\xi)/\gamma_0\right]d\xi'', \qquad (73)$$

where $\Phi_2 = -\xi\Delta\theta(L_s/L_f) + y^2/2(L_f + L_s)$, $\Psi_2 = (1/2F)(L_s/L_f)(\xi''-\xi)^2 + \Delta\theta(1 + L_s/L_f)\xi''$. In the form (72) for the amplitude, the point ξ''' of the first plate is imaged to the observation point $\xi = -(L_f/L_s)\xi'''$ of the focusing plane, while in the form (73) the point ξ'' of the second plate is imaged to the point $\xi = \xi''$. Formulae (72) and (73) are equivalent. The intensity in the focusing plane ($L = L_f$) is represented as

$$I_f = \left|E^e_{0h}\right|^2/|E|^2, \qquad (74)$$

where $E = \exp[ik(L_f + L_s)]/(L_f + L_s)$ is the amplitude of the point-source wave at the distance (L_f + L_s) without any focusing or diffracting system in the path of the wave. This representation provides the possibility to not only describe the intensity distribution but also to find the intensity increase in comparison with the intensity from a point source at the same distance (gain). From the obtained formulae it is obvious that

I_f is a function of ξ and L_f/L_s. It is easy to see that L_f/L_s is the magnification factor of the RDL. Let us consider two point sources placed at a distance L_s from the RDL. If $|\Delta\xi_s|$ is the distance between the two points and the glancing angles of waves emitted from these two points are $(\theta_1 - \alpha)$ and $(\theta_2 - \alpha)$, then for each point the double-diffracted wave with the surface of the second plate forms the glancing angle $(\theta_1 - \alpha)$ and $(\theta_2 - \alpha)$. Therefore at the distance $(L_{f1} + L_{f2})/2 \simeq L_f$, the reversed image of these two points is formed. It is obvious that $|\theta_1 - \theta_2| L_f = |\Delta\xi_f|$, where $\Delta\xi_f$ is the distance between the images of the point sources. The magnification M is determined as

$$M = \left|\Delta\xi_f\right| / \left|\Delta\xi_s\right| = L_f/L_s. \tag{75}$$

This result is obvious also from expression (72), because the maximum value of the function $G_{0h}[(\xi + \xi''' L_f/L_s)/\gamma_0]$ takes place at the point $\xi = -\xi'''(L_s/L_f)$. Introducing M into (73),

$$\begin{aligned} I_f = {} & (1+M)^2 (2\pi F/k) \\ & \times \left| \int \exp(\Psi_3) \frac{J_1(\sigma\xi''/\gamma_0)}{\xi''} \, \frac{J_1[\sigma(\xi''-\xi)/M\gamma_0]}{\xi''-\xi} \right. \\ & \left. \times \, H(\xi''-\xi) H(\xi'') \, d\xi'' \right|^2, \end{aligned} \tag{76}$$

where $\Psi_3 = ik[(1/2FM)(\xi''-\xi)^2 + \xi''\Delta\theta_c(1+1/M)] - \xi''\sigma_{0i}/\gamma_0 - (\xi''-\xi)\sigma_{0i}/M\gamma_0$, σ_{0i} is the imaginary part of σ_0, and $\Delta\theta_c = \theta - \theta_c$ is the deviation from the Bragg-corrected angle $\theta_c = \theta_0 - \sigma_{0r}/k\gamma_0$, where σ_{0r} is the real part of σ_0.

Based on (76), by numerical integration for various M and for $\lambda = \lambda_m$ (λ_m is the wavelength corresponding to the maximum amplitude component in the incident wave) and $\Delta\theta_c(\lambda_m) = 0$, the intensity distribution on the focusing plane $L = L_f$ is calculated. Let us take a lens made of beryllium. For the Si(220) Mo $K\alpha_1$ reflection (λ_m = 0.709 Å) the intensity distributions for various values of M are shown in Fig. 8. The refractive index of a material is represented as $n = 1 - \delta - i\beta$. In the case under consideration, $\delta = 1.118 \times 10^{-6}$ and $\beta = -2.69 \times 10^{-10}$, $\theta_0 = 10.626°$, $\alpha = 9.626°$, $\gamma_0 = 0.0175$, $\gamma_h = 0.346$, $b = 0.05$, $F_0 = R/2\delta = 448.43$ m, $F = F_0 b^2 = 1.14$ m. It is supposed that the radius of the lens $R = 1$ mm, $T_0 = 0.1$ mm and the aperture of the lens is $2R_0 > 2$ mm. $\mu T_{max} = (4\pi|\beta|/\lambda)T_{max} \geq 0.05 \ll 1$ can be estimated, where μ is the linear absorption coefficient of the lens and T_{max} is the maximal thickness of the lens. Therefore, the formula (76) can be used where the absorption of the lens is neglected. In (76), from $\Delta\theta_c$ one can pass to $\Delta\lambda = \lambda - \lambda_m$ taking into account that

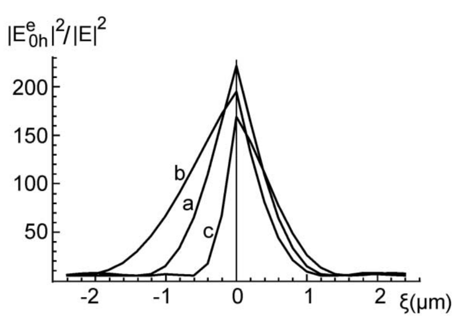

Figure 8
The double-diffracted beam intensity distribution on the focusing plane in a RDL with a beryllium lens. Shown are the cases corresponding to magnifications $M = 1$ (*a*), $M = 1.5$ (*b*), $M = 0.5$ (*c*). The beam falls at the Bragg-corrected angle corresponding to the wave with $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is neglected. Numerical integration by formula (76). Point source.

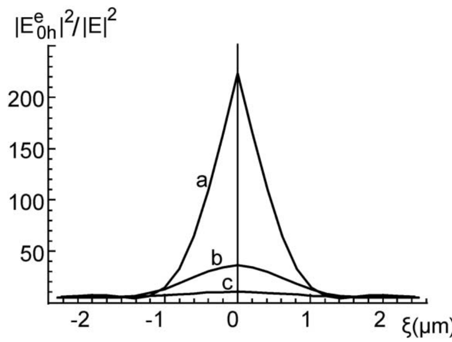

Figure 9
The double-diffracted beam intensity distribution on the focusing plane in a RDL with a beryllium lens. Shown are the cases corresponding to $\Delta\lambda/\lambda_m = 0$ (*a*), $\Delta\lambda/\lambda_m = 1.5 \times 10^{-4}$ (*b*), $\Delta\lambda/\lambda_m = -1.9 \times 10^{-4}$ (*c*). The beam falls at the Bragg-corrected angle corresponding to the wave with $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is neglected. $M = 1$. Numerical integration by formula (76). Point source.

$$\Delta\theta_c = \Delta\theta_c(\lambda_m) - (\Delta\lambda/\lambda_m)\tan\theta_0. \tag{77}$$

Supposing that $\Delta\theta_c(\lambda_m) = \theta - \theta_c(\lambda_m) = 0$ and inserting (77) into (76), one can refer to the $\Delta\lambda$ dependence of the intensity. Using the obtained expression and numerical integration, the intensity distributions for various $\Delta\lambda/\lambda_m$ for the case $M = 1$ are shown in Fig. 9. The parameters of the reflection and the lens in Figs. 9 and 8 are the same.

5.2. Absorptive lens

For an absorptive lens, $\chi_{0l} = \chi_{0lr} + i\chi_{0li}$ is a complex number. In this case the intensity distribution not only on the focusing plane but also along the focusing line must be studied. In (70), $B_1 = k(1/L - 1/L_f + i\chi_{0li}/Rb^2)/2$. Denoting

$$\Delta = 1/L - 1/L_f \tag{78}$$

and using the relation $\chi_{0li} = -2\beta$, B_1 can be represented as $B_1 = k(i\Delta - |\beta|/F\delta)/2$. For a large lens, making the integration in (70) the amplitude is written as

$$\begin{aligned} E^e_{0h}(\mathbf{r}) = {} & A_a \int \exp(\Phi_a) \left[J_1(\sigma\xi''/\gamma_0)/\xi''\right] \left[J_1(\sigma\xi'''/\gamma_0)/\xi'''\right] \\ & \times H(\xi'') H(\xi''') \, d\xi'' \, d\xi''', \end{aligned} \tag{79}$$

where

$$A_a = \exp(i\phi)\left\{F/\left[LL_s(L+L_s)\left(|\beta/\delta| - i\Delta F\right)\right]\right\},$$

$$\phi = kL_s + k\chi_{0l}T_0/2 - \pi/4 + ky^2/2(L+L_s),$$

$$\begin{aligned} \Phi_a = {} & ik\left[(\xi - \xi'')^2/L + \xi'''^2/L_s\right] \\ & + ik\Delta\theta_c(\xi'' + \xi''') - \sigma_{0i}(\xi'' + \xi''')/\gamma_0 \\ & - kF\left[\xi'''/L_s - (\xi'' - \xi)/L\right]^2 / 2\left(|\beta/\delta| - iF\Delta\right). \end{aligned}$$

In the focusing plane, $\Delta = 0$. Note that, in the X-ray range of frequencies, $|\beta/\delta| \ll 1$; therefore, the focusing distance L_f does not depend on β. The exponent $\exp\{-kF[\xi'''/L_s - (\xi'' - \xi)/L]^2/2(|\beta/\delta| - iF\Delta)\}$ can be written as $\exp\{-kF(|\beta/\delta| + iF\Delta)[\xi'''/L_s - (\xi'' - \xi)/L]^2/2(|\beta/\delta|^2 + F^2\Delta^2)\}$. If $\Delta = 0$, this exponent is like a δ-function and the functions under the integral sign give contributions near the point $\xi'''/L_s = (\xi'' - \xi)/L$. The half-width of this exponent increases with increasing $|\Delta|$ and oscillations also take place. Thus the integral has a small value. If $F|\Delta| \simeq |\beta|/\delta$ and therefore $|\Delta| \simeq (|\beta|/\delta)L_f^2/F$, then the value of the integral is close to its maximal value. In this way the longitudinal size of the focus point can be approximately estimated. The transverse size of the focus is determined from the half-width of the point-source functions G_{0h} and G_{h0} which figurate under the signs of the integrals (72) or (73). Using (72), $|\Delta\xi'''| \simeq \gamma_0/\sigma_r$ and therefore $|\Delta\xi_f| \simeq (1+M)|\Delta\xi'''| \simeq (1+M)\gamma_0/\sigma_r$. The value $|\Delta\xi_f| \simeq \gamma_0/\sigma_r$ of the focus size is the minimal value achieved in the case of the incident plane wave.

As in formula (76), one can refer to the $\Delta\lambda$ dependence in formula (79) using the relation $\Delta\theta_c = -(\Delta\lambda/\lambda_m)\tan\theta_0$ when $\Delta\theta_c(\lambda_m) = 0$. In this case in (79)

$$\begin{aligned}\Phi_a = {} & ik\Big[(\xi - \xi'')^2/L + \xi'''^2/L_s\Big] - ik\tan\theta_0(\Delta\lambda/\lambda_m)(\xi'' + \xi''') \\ & - \sigma_{0i}(\xi'' + \xi''')/\gamma_0 \\ & - kF\big[\xi'''/L_s - (\xi'' - \xi)/L\big]^2/2\big(|\beta/\delta| - iF\Delta\big).\end{aligned}$$

By numerical integration of (79), in Fig. 10 the intensity distributions on the focusing plane for various M and $\Delta\lambda = 0$, $\Delta = 0$, $\Delta\theta_c = 0$ are shown. The intensity distributions on the focusing plane for various $\Delta\lambda$ and $M = 1$, $\Delta\theta_c(\lambda_m) = 0$ are also shown in Fig. 11. The intensity distribution on the focusing line and its dependence on L for $\Delta\lambda = 0$ is given in Fig. 12.

All curves are given for the Si(220) Mo $K\alpha_1$ reflection and for a lens made of silicon. The incident wave is σ-polarized, $R = 1$ mm, $T_0 = 0.1$ mm. The aperture of the lens $R_0 \geq 2$ mm as in the case of the RDL made of beryllium. However, now $F = 0.8$ m, $F_0 = R/2\delta = 316.26$ m.

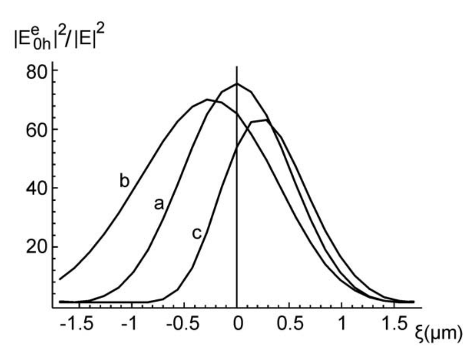

Figure 10
The double-diffracted beam intensity distribution on the focusing plane in a RDL with a silicon lens. Shown are the cases corresponding to magnifications $M = 1$ (*a*), $M = 1.5$ (*b*), $M = 0.5$ (*c*). The beam falls at the Bragg-corrected angle corresponding to the wave with $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is taken into account. Numerical integration by formula (79). Point source.

5.3. Spatial resolution

Let us now compare the spatial resolution of a RDL with the spatial resolution of an individual cylindrical lens. Note that comparison with the compound lens is not informative because the compound lens with 400 lenses ($1/b^2 = 400$) has a longitudinal size of more than 0.4 m and is not of interest practically. For a cylindrical non-absorptive lens, as described in the plane-wave section, the intensity distribution in the focusing plane is given by

$$I_{fc} = 4R_0^2\big[\sin(kxR_0/L_f)/(kxR_0/L_f)\big]^2(kF/2\pi)/L_fL_s.$$

According to the Rayleigh criterion, two point sources are considered to be resolved if $\Delta x_f = \pi L_f/kR_0$. However, $\Delta x_s = \Delta x_f L_s/L_f = \pi L_s/kR_0$. This is the resolution of a cylindrical lens. One can estimate that $\Delta x_s \geq 15$ µm. For this estimation, the values $L_s \geq 500$ m, $R_0 = 1$ mm, $k = 10^{11}$ m^{-1} (Mo $K\alpha$ radiation) are taken. In the case of the RDL, according to (72) and the Rayleigh criterion, two points are resolved if $\Delta\xi_f = 3.8(1+M)\gamma_0/\sigma_r$ and $\Delta\xi_s = \Delta\xi_f L_s/L_f = 3.8[(1+M)/M]\gamma_0/\sigma_r$. For this estimate the first zeros of the functions G_{0h} and G_{h0} in (72) are used. For the case under consideration, $M > 1$ and $\Delta\xi_s \simeq 3.8\gamma_0/\sigma_r$ can be used. For the case of a non-absorptive

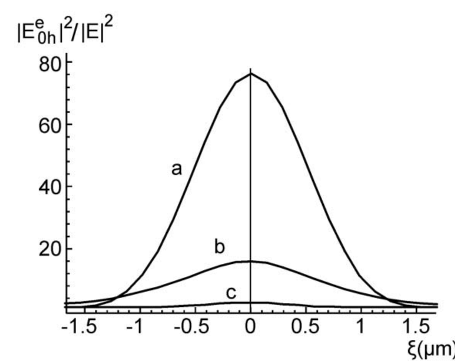

Figure 11
The double-diffracted beam intensity distribution on the focusing plane in a RDL with a silicon lens. Shown are the cases corresponding to $\Delta\lambda/\lambda_m = 0$ (*a*), $\Delta\lambda/\lambda_m = 1.5 \times 10^{-4}$ (*b*), $\Delta\lambda/\lambda_m = -1.9 \times 10^{-4}$ (*c*). The beam falls at the Bragg-corrected angle corresponding to the wave with $\Delta\lambda/\lambda_m = 0$. Absorption in the lens is taken into account. Numerical integration by formula (79). Point source.

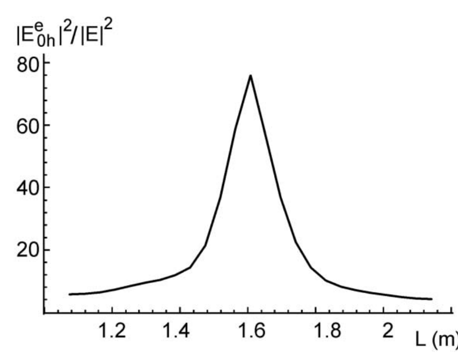

Figure 12
The double-diffracted beam intensity distribution on the focusing line in a RDL with a silicon lens. $M = 1$, $\Delta\lambda/\lambda_m = 0$, $\Delta\theta_c(\lambda_m) = 0$. Absorption in the lens is taken into account. Numerical integration by formula (79). Point source.

lens made of beryllium, for Si220 (Mo $K\alpha$) radiation and γ_0 = 0.02 one can estimate $\Delta\xi_s$ = 1.8 μm, *i.e.* approximately ten times smaller than for the cylindrical lens with the same parameters. Here note also that with increasing M the resolution of the RDL stays almost unchanged.

6. Conclusions

The results obtained in this paper can be summarized as follows.

(i) A one-dimensional focusing X-ray element is proposed. This element includes two plane parallel asymmetric-cut crystalline plates and a cylindrical parabolic double-concave lens placed in the gap between the plates.

(ii) Using the geometric optical method it is shown that the focal distance of this element is equal to the focal distance of a separately taken lens multipled by the square of the asymmetry factor: $F = F_0 b^2$.

(iii) If $b < 1$, then $F < F_0$. A practically useful focal distance can be achieved by taking an appropriate b. For example, when $F_0 \simeq 400$ m and $b = 0.05$, then $F \simeq 1$ m. In this sense the RDL is equivalent to an X-ray compound lens with 400 lenses.

(iv) The focus distance of a point source is determined by the well known thin lens formula $1/L_s + 1/L_f = 1/F$. For two point sources the reversed image is formed. The magnification M is determined by the well known optics expression for a thin lens $M = L_f/L_s$.

(v) In paraxial approximation the focus point does not depend on wavelength. The focus distance and focus coordinate on the focusing plane are the same for all wavelengths.

(vi) A one-dimensional focusing RDL can be upgraded to a two-dimensional focusing element. The use of two RDLs, each of which focus the beam perpendicular to each other, can realise two-dimensional focusing. The diffraction vectors $\mathbf{h}$ of the RDLs must lie in the $(\mathbf{K}, \mathbf{e}_z)$ and $(\mathbf{K}, \mathbf{e}_y)$ planes. Here $\mathbf{K}$ is the wavevector of the incident wave, $\mathbf{e}_y$ is the unit vector, perpendicular to the plane XOZ. The focus distance and magnification are defined by the same formulae as described in (iv).

(vii) For determination of the longitudinal and transverse sizes of the focus spot, the intensity distribution on the focusing plane and on the focusing line, and for describing the imaging of objects, a wave optical theory is necessary. The wave optical theory for the amplitude of the double-diffracted beam is given when an arbitrary wave falls on the system.

(viii) This theory is applied for two cases: plane-wave focusing and imaging of point sources. The formulae obtained confirm the results (iv) and (v). In addition, it is shown that the focus size is $\sim$1 μm. The angular and spatial resolutions are given and compared with the resolution of a separately taken lens.

(ix) The intensity at the point of observation can increase by two orders.

(x) The losses are determined by absorption in the lens and in the plates. Besides, the lens can affect the Bragg reflection from the second plate. The analysis of the transmission coefficient and its dependence on wavelength and other parameters is given. It is shown that waves with wavelengths $|\Delta\lambda|/\lambda \simeq 10^{-4}$ can effectively pass through the RDL. The transmission of the RDL can be close to unity. Note that the transmission can be up to 0.3 for a compound lens.

(xi) RDLs can be used in X-ray astronomy for imaging of objects and in X-ray microscopy. They can be used as a collimator if a point source is placed at the distance F from the RDL.

In the future it is planned to continue the investigation of RDLs, particularly the influence of the finite size of the lens on the focusing, and object imaging will be considered.

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