New figuring model based on surface slope profile for grazing-incidence reflective optics

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Surface slope profile is widely used in the metrology of grazing-incidence reflective optics instead of surface height profile. Nevertheless, the theoretical and experimental model currently used in deterministic optical figuring processes is based on surface height, not on surface slope. This means that the raw slope profile data from metrology need to be converted to height profile to perform the current height-based figuring processes. The inevitable measurement noise in the raw slope data will introduce significant cumulative error in the resultant height profiles. As a consequence, this conversion will degrade the determinism of the figuring processes, and will have an impact on the ultimate surface figuring results. To overcome this problem, an innovative figuring model is proposed, which directly uses the raw slope profile data instead of the usual height data as input for the deterministic process. In this paper, first the influence of the measurement noise on the resultant height profile is analyzed, and then a new model is presented; finally a demonstration experiment is carried out using a one-dimensional ion beam figuring process to demonstrate the validity of our approach.

1. Introduction

Surface slope is widely used in the metrology of grazing-incidence reflective optics instead of surface height (Idir *et al.*, 2014; Qian *et al.*, 2015; Siewert *et al.*, 2004, 2012, 2014). The wide use of surface slope benefits from the two following factors: (i) for most grazing-incidence reflective optics, a slope profile can better specify the optical performances than a height profile (Danjon & Couder, 1990); (ii) a slope profile is relatively easier to measure than a height profile for most grazing-incidence reflective optics. Slope profilers are more 'universal' test systems and do not require null configuration. Interferometers are limited in aperture, and grazing-incidence reflective optics with long size, or with steep curvature, are difficult to measure with classical interferometers. Therefore, most grazing-incidence reflective optics are measured by slope measuring systems, such as long trace profiler (LTP) (Qian & Takacs, 2007; Thomasset *et al.*, 2005), nanometer optical machine (NOM) (Alcock *et al.*, 2010; Siewert *et al.*, 2014) or 2D stitching Shack Hartmann optical head (SSH-OH) (Idir *et al.*, 2014). The output data of all these slope measuring systems are one-dimensional (1D) slope profiles or two-dimensional slope maps.

However, the current figuring model used in the deterministic fabrication is based on the surface height profile. We called it the height-based figuring (HF) model. In the HF model, the height amount removed by a figuring process is the
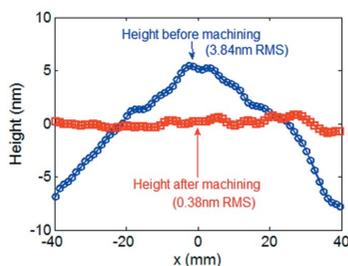
convolution of the height removal function and the dwell time of the process (Drueding *et al.*, 1995). To perform a figuring process with slope measuring systems, the original output, raw slope profile data, need to be converted by integration into height profile. Because the raw slope profile data inevitably contain measurement noise, although the noise can be very low in the slope domain, it will introduce a significant accumulation error in the resultant height profile (Artemiev *et al.*, 2012). We will illustrate this issue in the following section. The introduced accumulation error will degrade the determinism of the figuring processes and will have an inevitable impact on the ultimate surface figuring results. In addition, the conversion of slope data to height data will always cause a shift of the positioning of the topography with respect to the sample coordinates.

In our previous works (Zhou *et al.*, 2016), using the traditional HF mode and a slope measuring system (Idir *et al.*, 2014), we had achieved correction accuracies on grazing-incidence reflective optics below 1 nm RMS. But it was very difficult to reach below 0.5 nm RMS, although both the ion beam figuring (IBF) process and the slope metrology have excellent repeatability. We notice that the introduced accumulation errors from the conversion of slope to height may cause the problem and limit further accuracy. Peverini *et al.*, in their work to profile aspherical X-ray mirrors with a stationary broad ion beam and two movable blades, used slope profiles instead of height profiles to calculate the moving speed of the blades, and obtained better results (Peverini *et al.*, 2010). However, their method and discussion are only limited to their particular case. In our study, we consider this problem in a more general way and have developed a new figuring model, which uses the raw slope profile data instead of the converted height profile data. In this new slope-based figuring (SF) model, because the process calculations use directly the raw slope data, the accumulation errors from the conversion of slope to height can be avoided, and high convergence rate and high accuracy can be more easily achieved.

In the following section of this paper, the accumulation error introduced by the conversion from slope to height is analyzed, then the new SF model is presented, and a demonstration experiment carried out using a 1D IBF process to demonstrate the validity of our approach.

2. Error analysis

Consider a set of slope measurement data $s_1, s_2, s_3, \ldots, s_n$ for positions $x_1, x_2, x_3, \ldots, x_n$ with an increment length d. The simplest way to convert the slope data into height data is to apply a Riemann summation,

$$h_k = d \sum_{i=1}^{k} s_i. \tag{1}$$

However, in any measurement the measuring error is inevitable. In this case, if the slope measurement at position x_i has an error ε_i, then the corresponding height error at position x_k is

$$e_k = d \sum_{i=1}^{k} \varepsilon_i. \tag{2}$$

If we assume the slope errors to be random, independent and identically distributed, with zero mean and variance σ^2, *i.e.* $\varepsilon_i \sim N(0, \sigma^2)$, according to the random theory, the resultant height error at position x_k is subject to a k-dimensional Gaussian distribution

$$e_k \sim N(0, kd^2\sigma^2). \tag{3}$$

This means that in the converted height data the uncertainty at the kth point will be enlarged k times compared with the slope data. Artemiev *et al.* (2012) noted that the trapezoid rule, Simpson's rule and other Newton–Cotes approximate integration techniques suffer essentially the same drawback as the Riemann summation, even the advanced discrete Fourier transform method (Campos *et al.*, 2002); the uncertainty in the height data is significantly enlarged as well.

Here, we illustrate the influence of the noise in the slope profile data on the converted height profiles by MATLAB® simulation. Considering a 250 mm-long mirror, with 1.2 mm measuring step, and a noise distribution variance of 10 nrad, two noise sequences (slope profile errors, in units of μrad) generated by MATLAB® command "0.01*randn(1,208)" are shown in Figs. 1(*a*) and 1(*b*) by solid blue lines. According to equation (2), the cumulative sum of each sequence, multiplied by a factor of 1.2 (measuring step, mm), can be derived to the height profile errors (in nm), which are shown in Fig. 1 by green dotted lines.
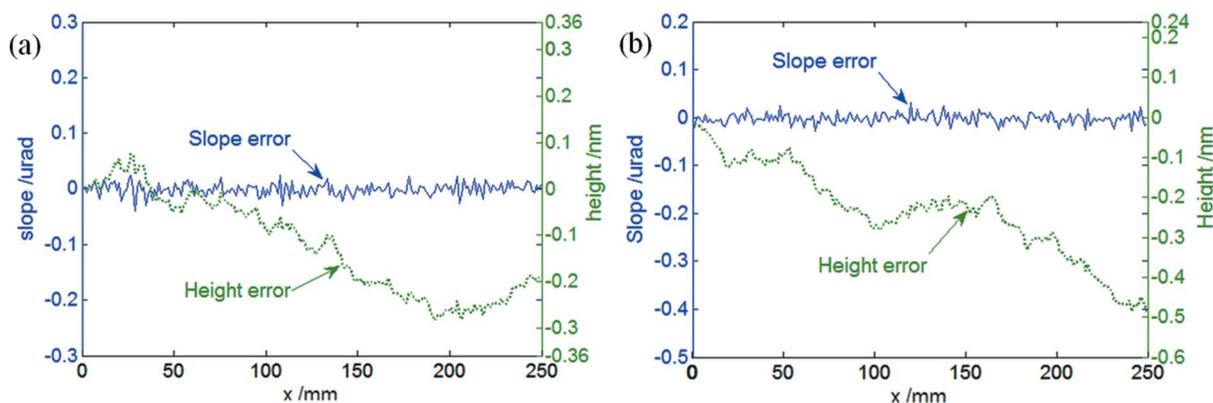

Figure 1
Two examples illustrating the influence of the slope noise on the converted height profile.

The simulation results show that, although the measurement noises are as small as 10 nrad, the converted height errors are in the level of 0.2–0.3 nm for a 250 mm length, and can sometimes be even greater than 0.4 nm. These errors are too big to be neglected in the testing and figuring of X-ray diffraction-limited mirrors, where the acceptable height errors must be below ($\lambda/27\theta$) r.m.s. ($\sim$0.6 nm r.m.s., for λ = 0.05 nm and θ = 3 mrad) (Pardini *et al.*, 2015). Since all the calculations in the conversion from slope to height are linear, if the measurement noise in the slope profile increases, the converted height error will be linearly increased. Furthermore, with the increase of the length of the mirror, the number of measurement points will be increased, and the converted height error will be linearly increased as well.

In brief, although the measurement noises in slope profile data are very small in slope domain, the introduced error in height domain may be significant enough to affect the determinism of figuring ultra-precision X-ray diffraction-limited mirrors.

3. Slope-based figuring model

In the traditional HF model, the height amount removed by a figuring process is the convolution of the height removal function and the dwell time of the process. Here, we only discuss the 1D case

$$r(x) = \int_{-\infty}^{+\infty} p(x - u)\tau(u)\,\mathrm{d}u \ = \ p(x) \otimes \tau(x), \qquad (4)$$

where $r(x)$ is the height amount removed by the process, $p(x)$ is the height removal function of the process, $\tau(x)$ is the dwell time in the process, and $\otimes$ stands for the convolution operation. By differentiating (4),

$$\frac{\mathrm{d}r(x)}{\mathrm{d}x} = \frac{\mathrm{d}\int_{-\infty}^{+\infty} p(x-u)\tau(u)\,\mathrm{d}u}{\mathrm{d}x}$$
$$= \frac{\int_{-\infty}^{+\infty} \mathrm{d}p(x-u)\tau(u)\,\mathrm{d}u}{\mathrm{d}x}, \qquad (5)$$

we obtain

$$s(x) = \int_{-\infty}^{+\infty} q(x - u)\tau(u)\,\mathrm{d}u \ = \ q(x) \otimes \tau(x), \qquad (6)$$

where $s(x) = \mathrm{d}r(x)/\mathrm{d}x$ is the derivative of the height $r(x)$, *i.e.* the slope; $q(x) = \mathrm{d}p(x)/\mathrm{d}x$ is the derivative of the height removal function, which we refer to as the slope removal function.

Equation (6) is the model used to describe a deterministic optical figuring process based on slope profile. It reveals that the slope amount removed by a figuring process is the convolution of the slope removal function and the dwell time in the process. This general formula can be used in other applications where slope profiles are of more interest than height.

According to (6), a deconvolution process is necessary to solve the dwell time $\tau(x)$ from the slope profile error $s(x)$ and the slope removal function $q(x)$. This calculation is the same as using equation (4) to solve the dwell time. Therefore, most of the deconvolution algorithms (Carnal *et al.*, 1992; Drueding *et al.*, 1995; Zhou *et al.*, 2007; Liao *et al.*, 2014) applied in the HF model can be used in the new SF model.

With this new SF method, the slope profile data do not have to be converted into height profile data. Therefore, the introduced accumulation error from the conversion can be avoided and the determinism of the figuring processes can be improved.

4. Experimental results and discussion

Experiments were carried out on a 1D IBF system developed at Brookhaven National Laboratory (BNL) (Zhou *et al.*, 2016). Fig. 2(a) shows the obtained height removal function. By differentiation, we can find the slope removal function, as shown in Fig. 2(b).

The shape of the slope removal function appears to be 'strange' compared with the traditional height removal function, but this is not an issue for a deterministic optical figuring process. The only mandatory property for a removal function of a deterministic figuring process is stability. The matrix-based algorithm (Carnal *et al.*, 1992; Zhou *et al.*, 2007) can deal with this 'strange' removal function. We use this algorithm to solve the dwell time in our 1D IBF process.

The parameters in our figuring demonstration experiment are the same as those in the experiment to obtain the removal function. The sample in this demonstration is a plane silicon
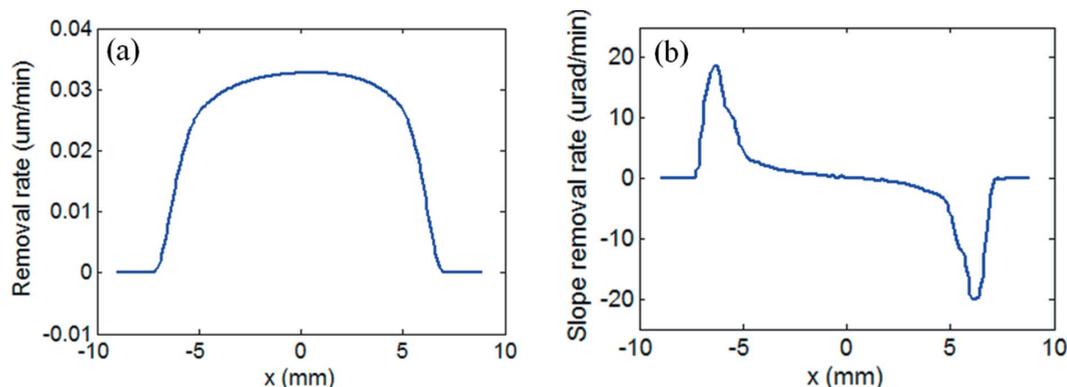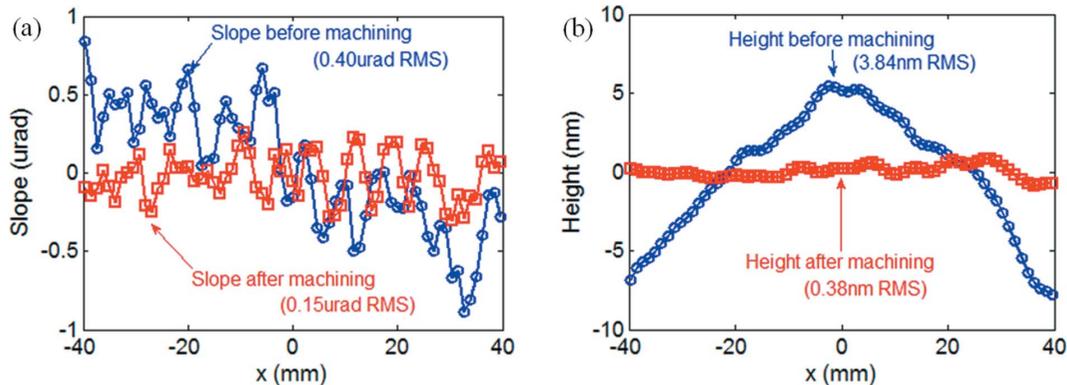

Figure 2
Removal functions in the 1D IBF process. (*a*) Height removal function. (*b*) Slope removal function.

Figure 3
Experimental result of 1D IBF using the SF method. (*a*) Slope profile error. (*b*) Height profile error.

mirror. After 3.3 min of the 1D IBF process, the surface slope profile error on the central line of the sample was reduced from 0.40 µrad RMS to 0.15 µrad RMS (see Fig. 3*a*). The surface slope was measured by a slope measuring instrument developed at BNL (Idir *et al.*, 2014). The corresponding height profile error was reduced by a factor of ten from 3.84 nm RMS to 0.38 nm RMS (see Fig. 3*b*). This experimental result shows that the deterministic figuring process driven by slope metrology data is very successful. Here, the height profile errors shown in Fig. 3(*b*) are only used to illustrate how the surface profile is corrected, but those height profiles are neither calculated nor used in our figuring process.

5. Conclusion

Due to the influence of measuring noise, the resulting height profile converted from raw slope data is imprecise. Subject to a Wiener random sequence, the error uncertainty at the kth point at the converted height profile will expand k times than that of the raw slope profile. The traditional HF model based on the converted height profile is therefore not suitable for the processes with raw slope data from slope measuring systems. We proposed a SF model which uses the raw slope data instead of converted height data to perform deterministic figuring. The SF model reveals that, in a deterministic optical figuring process, the slope amount removed by a figuring process is the convolution of the slope removal function and the dwell time in the process, where the slope removal function is the derivative of the height removal function. With this new SF model, the dwell time used to control the figuring process can be directly solved from the raw slope data, and the error-yield conversion from slope to height can be avoided. Therefore, a figuring process with the SF model can improve the determinism of the process and achieve a better result.

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