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The effect of beam time structure on counting detectors in SRS experiments

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Counting detector systems are increasingly used in X-ray experiments because of their attractive properties as regards linearity, large dynamic range and simple noise properties. In synchrotron radiation source (SRS) applications of X-ray detectors, counting rates are generally high enough to require dead-time correction. The time structure of an SRS beam interacts with the dead-time characteristics of the detector in a way that the simple stochastic dead-time models cannot always handle. This report generates analytical and Monte Carlo mathematical models which describe the rate performance of any given detector system when used with the typical beam structures encountered in an SRS.

Keywords: synchrotron radiation sources; X-rays; detectors; counting rates; dead time; beam structure.

1. Introduction

Counting detector systems are attractive for X-ray detection applications because of their inherent linearity, large potential dynamic range and well defined (statistical) noise characteristics. Their main limitation is the inevitable dead-time losses which appear at data rates comparable with the detector dead time. These dead-time losses are well understood when (as in the case of the detection of radioactive decay particles) the arrival of events is random in time (Knoll, 1989). Experiments on synchrotron radiation sources, on the other hand, combine very high data rates with a highly time-structured beam which complicates the simple stochastic model. The bunch structure of the electron beam in a synchrotron radiation source (SRS) produces an X-ray beam with the same time modulation. This time structure will (in general) interact with the intrinsic timing properties of any counting system used to detect the X-rays and can often limit the rate performance of the system.

Aspects of the problem have been discussed by Arndt (1978), Batterman (1980), Lee & Mills (1992), Cousins *et al.* (1993) and Cousins (1994). Using mathematical models, this report investigates more general formulations for assessing the performance of any given type of detector system when used with some typical SRS beam structures. The modelling approach required to describe the interaction between the beam structure and the detector system varies according to the relation between the counter dead time and the beam structure. It is convenient to consider the three distinct types of beam structure separately.

In the following the symbols N_{in} and N_{out} refer to the time-averaged rate of events (*i.e.* over a period long compared with any beam structure) which a detector system would deliver if it had no dead time (N_{in}) and the rate that it actually does deliver (N_{out}) in the context of a given beam structure. The two distinct types of dead-time model are denoted by the terms *counting above a simple discriminator* in which a single dead-time period is operative for each event and is not re-triggerable during that period, and *pulse height analysis* in which a clean spectrum demands rejection of pile-up events.

2. SRS beam structures

For the purposes of developing the models, we use the time structures used on the Daresbury Laboratory SRS. These are:

(i) *Flat fill.* The ring is full of evenly spaced bunches of electrons which give continuously an X-ray pulse of width 0.2 ns every 2 ns.

(ii) *Single bunch.* There is only one bunch in the ring which produces an X-ray pulse of width 0.2 ns every circuit of the ring, *i.e.* every 320 ns.

(iii) *Gapped beam.* In this case there is a gap in the bunch structure so that X-rays are delivered as in the flat-fill case for 200 ns of each beam circuit and are absent for the remaining 120 ns of the 320 ns period.

Different SRS machines will have a cycle time determined by the machine diameter (3 µs for the ESRF and ~1 µs for Diamond) with a similar fine structure and various gap patterns.

3. Dead-time modelling with a flat-fill beam

No counting detector in routine use can respond to the fine structure of the beam. The fastest counter in general use is probably a scintillator/PMT device which can operate with a dead time of a few tens of nanoseconds. Recently, avalanche photodiodes have become available with the potential to yield a detector dead time of a few nanoseconds, which is, however, still well beyond the single-bunch width [see Kishimoto (1997) for a discussion of this case]. The flat-fill case therefore reduces, as far as the counter system is concerned, to uniform delivery of X-rays in time. For this we can use the standard dead-time models. At any data capture rate accessible to the detector system there will be very much less than one detectable event per bunch so that the arrival of events will be effectively random in time.

3.1. *Counting above a simple discriminator*

In this case the response of the counting system (as a function of the input rate) is

$$N_{\rm out} = N_{\rm in}/(1 + N_{\rm in}\tau), \qquad (1)$$

where $N_{\rm out}$ is the data capture rate, $N_{\rm in}$ is the random X-ray rate driving the system and τ is the counting-system dead time which is usually a combination of intrinsic detector dead time and electronic dead time (Knoll, 1989).

In this case the data capture rate is 50% of $N_{\rm in}$ at $N_{\rm in} = 1/\tau$ and the data capture rate $N_{\rm out}$ asymptotes to $1/\tau$ as $N_{\rm in}$ tends to infinity. Fig. 1 illustrates the behaviour of equation (1) in terms of the natural parameters $N_{\rm in}\tau$ and $N_{\rm out}\tau$.

3.2. *Pulse height analysis*

In the case of pulse height analysis we require a clean signal free of pile-up distortion. This requires that a dead-time period must always be free of another event. Poisson statistics show that in this case

$$N_{\rm out} = N_{\rm in}\exp(-N_{\rm in}\tau).$$

This function peaks at a value of $N_{\rm in}/e$ at $N_{\rm in} = 1/\tau$ and declines to zero as $N_{\rm in}$ increases further. It is important to understand the significance of τ in this equation: it represents the sum of the occupancy times (the period in which the analogue pulse of the detector is significantly above threshold) of the pulse under consideration and any preceding pulse. If we wish to be consistent with the definition of τ in (1), it is necessary to write this function as

$$N_{\rm out} = N_{\rm in}\exp(-2N_{\rm in}\tau). \qquad (2)$$

This is the actual counting rate that will be observed if a pile-up rejector is fitted to the system. If one is not fitted then the detected rate will be given by (1), and (2) will give the rate of 'good' events in the PHA spectrum with $N_{\rm out}(1) - N_{\rm out}(2)$ events in the 'pile-up' spectrum. The word 'significantly' in the above definition of the occupancy time sets the limits to the acceptable distortion in the pulse-height spectrum; usually the occupancy time is defined as the period that the pulse is >1% of its amplitude above the base line. Fig. 1 illustrates the form of equation (2) graphically in terms of the natural parameters $N_{\rm in}\tau$ and $N_{\rm out}\tau$.

In conclusion we note that the rate performance of the detector systems is entirely governed by their own dead time and is not affected by the beam structure.

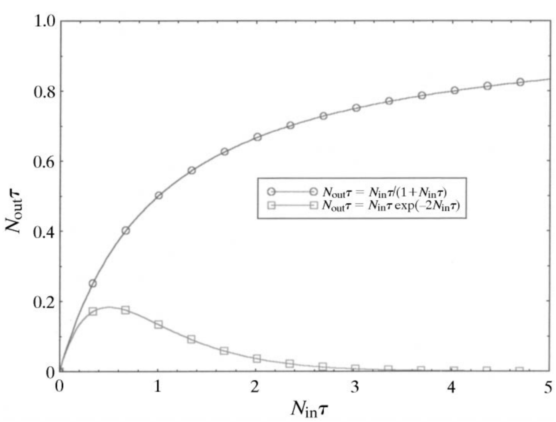

Figure 1
The characteristic throughput curve predicted by formula (1) for counting above a simple discriminator and by formula (2) for the uncorrupted rate in a pulse-height spectrum. The natural parameters $N_{\rm in}\tau$ and $N_{\rm out}\tau$ are used where $N_{\rm in}$ is the event rate hitting the detector and $N_{\rm out}$ is the data capture rate. τ represents the detector/electronic dead time in the first case and the event occupancy time in the second case. This is the case of random incidence with no detectable beam structure.

4. Dead-time modelling with a single-bunch beam

In this case the situation is dominated by the fact that all the X-rays emitted during a bunch crossing are inseparable (in time) by any normal detector system. The important statistic is therefore the number of X-rays (M) incident on the detector per beam pulse of width 0.2 ns. The following two distinct cases must be considered.

4.1. *Case 1: the detector dead time is shorter than the circulation period (τ_B)*

In this situation the detector registers one count only, irrespective of the value of M, but is ready to trigger on the arrival of the next beam pulse. In order to evaluate the number (m) of counts recorded we must consider the Poisson distribution of the number of X-ray counts (x) actually produced for a mean number of M,

$$P(x, M) = \exp(-M)M^x/x! \qquad (3)$$

where

$$\sum_x P(x, M) = 1.$$

The number of events at each occurrence number (x) is $xP(x, M)$, *i.e.* $\exp(-M)M^x/(x-1)!$, where

$$\sum_x xP(x, M) = M.$$

Summing the valid events we see that we can count the occurrences of single events, but double, treble *etc.* events only count as one,

$$m = 1P(1, M) + 1P(2, M) + 1P(3, M) + \ldots.$$

Using equation (3) we see that

$$\begin{aligned} m &= P(1, M) + \sum_{x2} P(2, M) \\ &= P(1, M) + [1 - P(0, M) - P(1, M)] \\ &= M\exp(-M) + 1 - \exp(-M) - M\exp(-M) \\ &= 1 - \exp(-M). \end{aligned}$$

Since we obtain M X-ray counts, of which we detect m at every beam pulse, our observed counting rate is

$$N_{\rm out} = [1 - \exp(-M)]/\tau_B. \quad (4)$$

If $N_{\rm in}$ is the rate of events hitting the detector then

$$N_{\rm in} = M/\tau_B$$

and we can reformulate (4) as

$$N_{\rm out} = [1 - \exp(-N_{\rm in}\tau_B)]/\tau_B. \quad (5)$$

As shown in Fig. 2, the data capture rate asymptotes to $1/\tau_B$ as the beam intensity ($N_{\rm in}$) is increased. This is the case in which we are simply counting above a simple discriminator.

When uncorrupted data (*i.e.* no pile-ups) is required then we are allowed to count only the term $P(1, M)$ in the Poisson distribution. This gives a detected rate of

$$\begin{aligned} N_{\rm out} &= M\exp(-M)/\tau_B, \\ &= N_{\rm in}\tau_B\exp(-N_{\rm in}\tau_B)/\tau_B \\ &= N_{\rm in}\exp(-N_{\rm in}\tau_B). \quad (5a) \end{aligned}$$

This is the same expression as (2) above with the difference that the dead time is the beam circulation period and, because the beam is synchronous in its temporal pattern, the factor of two is not present. This curve is also shown in Fig. 2.

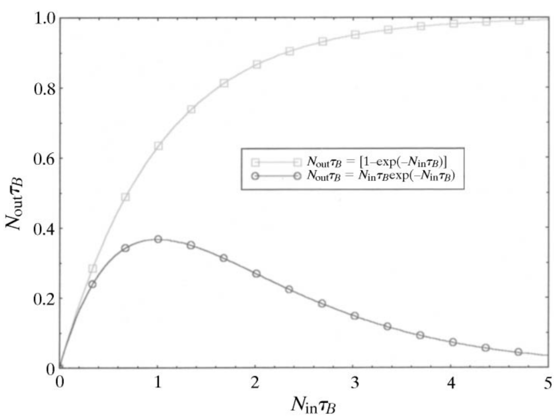

Figure 2
The corresponding curves generated by equations (5) and (5*a*) for the case of a single-bunch beam combined with a detector dead time which is less than the beam circulation period τ_B. It will be noted that the detector dead time is irrelevant if it is less than τ_B. In this figure, and in most of the subsequent figures, the natural parameters $N_{\rm in}\tau_B$ and $N_{\rm out}\tau_B$ are used.

In conclusion we note that the system throughput curves are very similar to the flat-fill case except that the governing time constant is now the beam circulation time. Thus counting above a simple discriminator, the ultimate rate of any counter is just the beam circulation frequency. The 'no pile-up' case is academic in this situation since it is impossible to make a pile-up rejector which can function within the 0.2 ns of the fine structure. However, the 'pile up' curve (Fig. 2) is of interest in that it shows the rate of uncorrupted events with the difference between the two curves showing the rate of 'piled up' events.

4.2. *Case 2: the detector dead time is longer than the beam circulation period*

In this case any bunches which follow one with a detected event will not be seen by the detector until the dead time of the detector has elapsed. We can apply the models derived above as follows.

Counting above a simple discriminator, the counting system sees a data capture rate governed by relation (1). In this case the input rate is controlled by the statistics of the countable events (m), *i.e.* relation (4). Thus,

$$N_{\rm out} = \frac{[1 - \exp(-M)]/\tau_B}{1 + [1 - \exp(-M)]\tau/\tau_B}.$$

By substituting $N_{\rm in} = M/\tau_B$, this becomes

$$N_{\rm out} = \frac{[1 - \exp(-N_{\rm in}\tau_B)]/\tau_B}{1 + [1 - \exp(-N_{\rm in}\tau_B)]\tau/\tau_B}.$$

To make this formula represent the true behaviour of the counting system we must observe that in the single-bunch mode the beam loss in the counter dead time is digitized to the number of bunch crossings covered by τ. Thus the final formula for this case becomes

$$N_{\rm out} = \frac{[1 - \exp(-N_{\rm in}\tau_B)]/\tau_B}{1 + [1 - \exp(-N_{\rm in}\tau_B)]n}, \quad (6)$$

where n is an integer defined by $n = {\rm Int}(\tau/\tau_B)$.

Checking the limiting cases of (6) we see that if $\tau/\tau_B < 1$ ($n = 0$) then (6) becomes (5) as required. If $\tau >> \tau_B$ and rates are high, then (6) limits to $1/\tau$, and if rates are low then (6) limits to (1).

Fig. 3 shows the input/output curves given by (6) for a few values of n. A rough rule of thumb emerges from Fig. 3 for very high rates ($>1/\tau_B$), namely that $N_{\rm out} \simeq \tau_B^{-1}(n+1)^{-1}$. As noted above, at low rates $N_{\rm out}$ tends to the standard formula (1) for $n >> 1$.

Similar reasoning can be applied to the pulse-height analysis case (*i.e.* no pile-ups allowed). Pile-up in this case arises in two distinct settings: pile-up within a bunch, the fraction of non-piled up events being

$$N_{\rm out}/N_{\rm in} = \exp(-N_{\rm in}\tau_B),$$

and pile-up in the detector system, where

$$N_{\rm out}/N_{\rm in} = \exp(-2N_{\rm in}\tau).$$

Combining these two fractions the overall rate of 'good' events is

$$N_{\rm out} = N_{\rm in}\exp(-N_{\rm in}\tau_B)\exp(-2N_{\rm out}\tau_B n)$$
$$= N_{\rm in}\exp[-N_{\rm in}\tau_B(2n+1)], \qquad (7)$$

where, as usual, $n = \mathrm{Int}(\tau/\tau_B)$. In the limiting case of $n = 0$ (no counter dead time) we see the formula for the bunch pile-up as expected, and at large detector dead times ($2n >> 1$) equation (7) reduces to the pile-up formula for the detector system alone, as expected.

Fig. 4 shows the behaviour of equation (7) for a range of values of n. In this figure the fraction $N_{\rm out}/N_{\rm in}$ denotes the fraction of 'clean' pulses in a pulse-height spectrum. Pile-up protection on the counter circuit can only remove the fraction generated by the counter time constant and the unresolvable pile-up within the bunch remains. This contribution to the pile-up will only become small at rates such that $N_{\rm in}\tau_B << 1$.

5. Dead-time modelling with a gapped beam

In the case of a gapped beam there are three interacting parameters describing the situation: the detector dead time (τ), the basic beam cycle time (τ_B) and the length of the 'on' time of the beam ($\tau_{\rm on}$). This leads to a situation which is analytically untractable, and to which the simplest modelling approach is a Monte Carlo computer program. The Monte Carlo model also provides the opportunity to check the analytical formulae developed for the simpler cases.

In order to compute the $N_{\rm out}$ *versus* $N_{\rm in}$ curves for the case of counting above a simple discriminator, we focus our attention on a large number of beam cycles (~1000) and generate events randomly in this interval building in the 'on' period. We express τ and $\tau_{\rm on}$ as fractions of τ_B and work in units of τ_B. In order to apply the dead time in the analogous fashion to an electronic system the chosen number of 'input' pulses (a few thousand) are sorted into ascending order and only counted into the 'output' total if they are more than one dead-time period from the preceding event. The input and output rates are expressed naturally as $N_{\rm in}\tau_B$ and $N_{\rm out}\tau_B$.

Fig. 5 shows the plots obtained from the program when $\tau_{\rm on}$ is set to the value appropriate for the SRS ($\tau_{\rm on} = 0.625\tau_B$) and the detector dead time is varied from $\tau = 0.1\tau_B$ to $10\tau_B$ (*i.e.* 32 nS to 3.2 µs corresponding to a scintillation detector at one end to a germanium detector at the other, for example). The input range of the counting rate is 0–6.25 MHz and the output range is 0–4.7 MHz.

The program can be modified in a simple manner to generate the corresponding curves for the case in which we require uncorrupted events (*i.e.* a clean pulse-height spectrum). The same logic is applied as above except that the dead-time period is extended forwards and backwards in time from the event under consideration.

Fig. 6 shows the $N_{\rm out}$ *versus* $N_{\rm in}$ curves generated in this way with the same parameter ranges as in Fig. 5. As expected, these curves do not asymptote to a maximum rate but peak at values of $N_{\rm in}$ and $N_{\rm out}$ inversely related to τ, but clearly not in any simple way.

Computation of the system throughput curves in terms of these generalized parameters means that the curves in

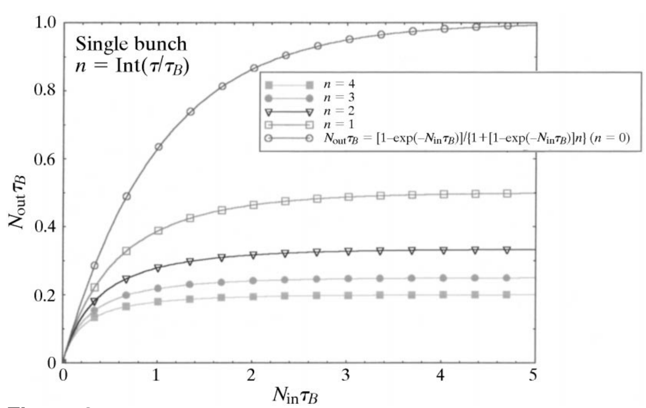

Figure 3
The predictions of equation (6) are plotted for various values of n [= $\mathrm{Int}(\tau/\tau_B)$] for counting above a simple discriminator with dead time longer than the beam circulation time in the case of a single-bunch beam.

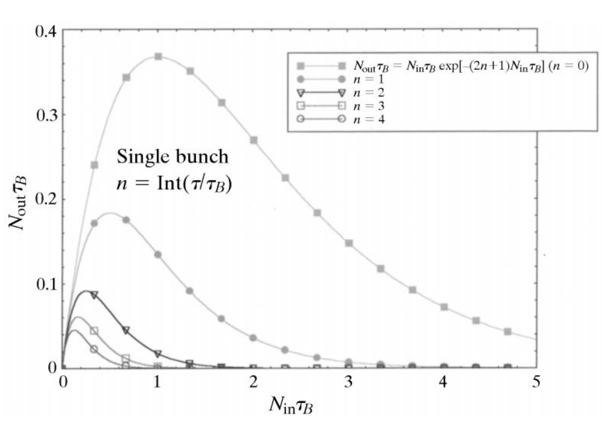

Figure 4
The equivalent predictions from equation (7) (pulse-height analysis) when the event occupancy is greater than the beam circulation time.

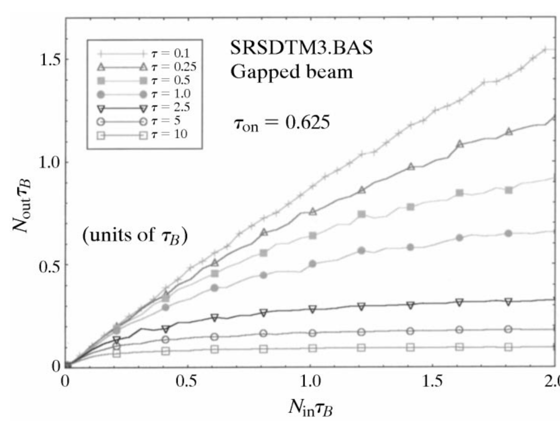

Figure 5
The throughput characteristics predicted by the Monte Carlo model for a gapped beam with the 'on' time ($\tau_{\rm on}$) set at the SRS value ($0.625\tau_B$) for counting above a simple discriminator with a range of detector dead times τ expressed as fractions of τ_B.

Figs. 5 and 6 can be used for any detector used with the given pattern of gapped beam. If a different τ_{on} is required then the curves must be recalculated. Any pattern of beam and gap can be modelled if it is required.

6. Comparison of the formulae with the Monte Carlo model

6.1. *Counting above a simple discriminator*

The simplest case of a flat-fill beam can be represented in the model by setting $\tau_{on} = 1.0$. If $\tau = 1$ (using units of τ_B) then the plot of $N_{out}\tau_B$ (y) *versus* $N_{in}\tau_B$ (x) is just $y = 1/(1+x)$ [equation (1)]. As Fig. 7 shows, the agreement between the model and the function is perfect within the statistical noise of the Monte Carlo model.

The model can represent a single-bunch beam by setting $\tau_{on} = 0.001$ (0.32 ns). Fig. 8 compares the output of the model with formulae (5) and (6). Again the agreement is perfect within the statistical error. Because the bunch is very narrow, the dead-time curves switch abruptly with the value of n. In reality, with detector time jitter the transitions will be less precise.

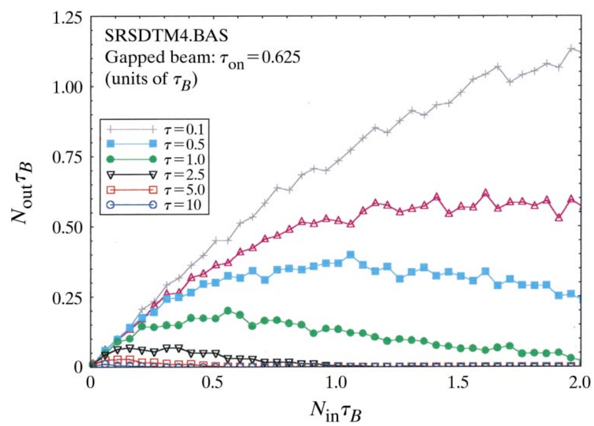

Figure 6
The corresponding model predictions for the 'clean' pulse-height analysis rates with a range of occupancy times.

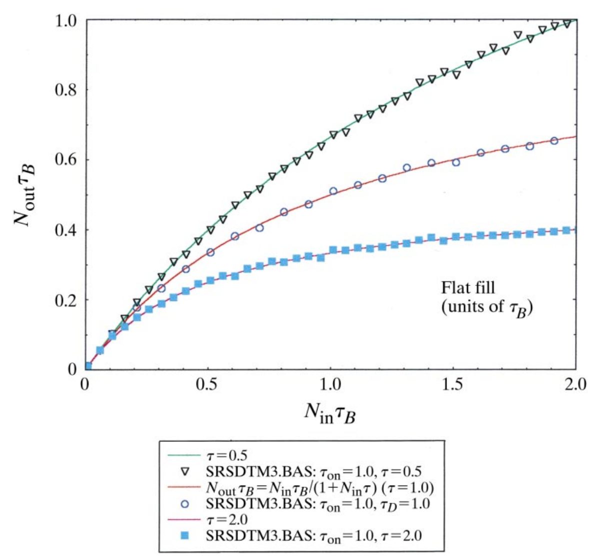

Figure 7
A comparison of the predictions of the Monte Carlo model in a flat-fill beam with equation (1) for the case of counting above a threshold. A range of dead times is shown.

6.2. *Pulse-height analysis*

Fig. 9 compares the rate of 'clean' events in the case of a flat-fill beam with τ in the range 0.5–2, as predicted by equation (2) and the Monte Carlo model.

Fig. 10 compares the predictions of formula (7) and the model in the case of a single-bunch beam when the detector

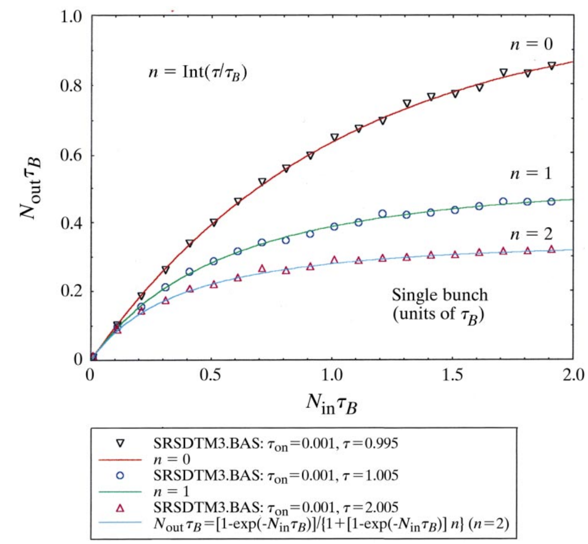

Figure 8
A comparison of the predictions of the Monte Carlo model in a single-bunch beam for the output rate (counting above a simple discriminator) with equation (6). A range of dead times is shown.

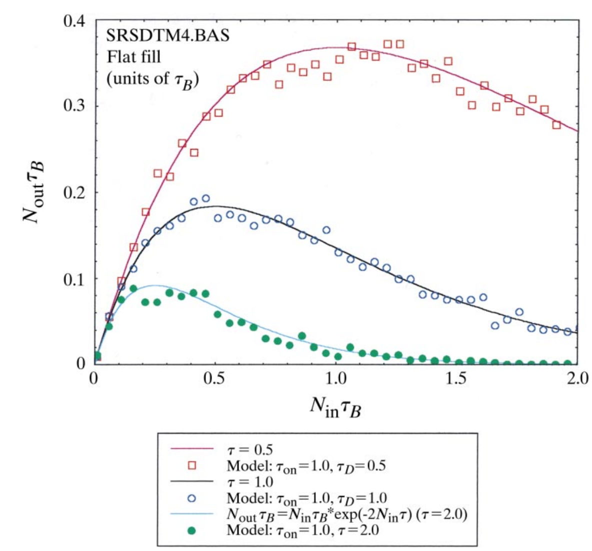

Figure 9
A comparison of the Monte Carlo model in a flat-fill beam ('clean' pulse-height analysis rate) with equation (2). A range of occupancy times is shown.

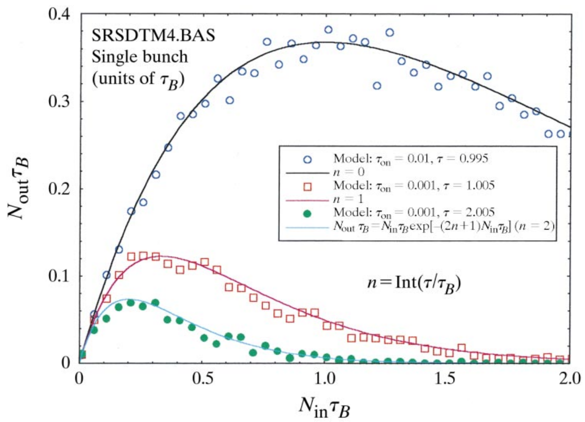

Figure 10
A comparison of the Monte Carlo model in a single-bunch beam ('clean' pulse-height analysis rate) with equation (7). A range of occupancy times is shown.

occupancy time increases from less than τ_B to greater than τ_B.

In all cases the agreement is perfect within the statistical noise of the Monte Carlo process. Since the Monte Carlo model simply mimics the corresponding electronic function, these results give confidence in the mathematical analysis (and *vice versa* in the case of the simplest formulae).

7. Conclusions

The models developed in this report permit detailed exploration of the performance of counting systems in any of the beam time structures generally met. As expected, counting above a simple discriminator leads to an asymptotic data capture rate and the rate of unpiled-up events peaks and thereafter declines. The main point to emerge is that in the flat-fill case the detector dead time (τ) is the controlling parameter and in the single-bunch case the beam circulation time (τ_B) is the controlling parameter. The result of the latter effect is that a detector with a dead time significantly less than τ_B will be limited to a rate of $\sim 1/\tau_B$. For such a detector the flat-fill case is more advantageous. For very slow detectors ($\tau >> \tau_B$) the difference is small.

There is one fast detector system which can significantly reduce the losses associated with single-bunch operation. Using a gas microstrip detector (GMD) with the electron drift direction normal to the beam direction (Bateman *et al.*, 1998, 2000) one can use the drift time of the X-ray-generated electron clouds to randomize the times of arrival at the detector anodes. The 320 ns period of the SRS corresponds to a drift distance of 15 mm, which is quite comfortable. For longer periods of up to several microseconds it is possible to choose slower drift conditions without compromising the basic speed of the anode pulse. Operating in this way the GMD can randomize the events in the beam spike and detect them with a limit set by its own dead time (~100 ns). On the SRS the gain in limiting rate is of the order of ×3 but, on the larger machines, factors of >10 could be achieved in single-bunch mode.

The performance of any detector system in gapped-beam mode is obviously intermediate between the single-bunch and flat-fill cases. The interaction between the parameters is complex and the practical method of solving it is to use the Monte Carlo model with the parameters of the situation appropriately inserted. The simple model used is approximate but adequate to elucidate the key features of the situation.

The models show clearly the great difficulty in obtaining clean pulse-height spectra (and clean discrimination) at high rates. As Figs. 6 and 10 show, if the occupancy time is larger than τ_B in single-bunch or gapped beams, the occupancy time of the detector must be less than τ_B to achieve a useful fraction of 'clean' data. Similarly, in a flat-fill beam (Fig. 9) the value of τ is critical to achieving 'clean' data at high rates.

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