J. Synchrotron Rad. (1998). **5**, 946–948

Mössbauer surface-guided waves in media with space modulation of Mössbauer isotope concentration

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(*Received 4 August 1997; accepted 26 January 1998*)

Rapid progress in Mössbauer spectroscopy combined with synchrotron radiation (SR) makes it urgent to study the interaction of SR with structures which have a space modulation of the Mössbauer isotope concentration, and thereby to reveal new effects in this interaction. Presented here are theoretical studies of the Mössbauer surface-guided mode (MSGM) in such media. MSGMs are analogous to the well known surface-guided electromagnetic waves (SGEW) in periodic media. However, because of their resonant interaction with Mössbauer nuclei they reveal some qualitative peculiarities compared with the case of conventional SGEW. The MSGMs for the case of purely nuclear Bragg scattering of quanta at a plane interface between a homogeneous medium and a medium with a periodic space modulation of the Mössbauer isotope concentration are investigated theoretically. The conditions of MSGM existence and allowed spectral intervals for the MSGM and their dependence on the period of space modulation and the degree of medium enrichment by the Mössbauer isotope are found. It is shown that the allowed interval of the MSGM frequencies is located at one side relative to the exact Mössbauer resonant frequency. Its width is of the order of the Mössbauer line width and can be effectively changed by the variation of the modulation period or the Mössbauer isotope concentration. For the MSGM frequencies close to the boundary of the MSGM existence interval, the attenuation of an MSGM propagating along the interface may be essentially lower than the conventional Mössbauer radiation attenuation in the same medium. The possibilities of SR Mössbauer filtration by means of MSGM are briefly discussed.

Keywords: Mössbauer surface waves; Mössbauer filtration.

1. Introduction

An actively developing branch of Mössbauer spectroscopy is the application of synchrotron radiation (SR) in investigations of the interaction of SR (Belyakov, 1992) with structures with a space-modulated content of a Mössbauer isotope (Chumakov *et al.*, 1992, 1993; Gusev *et al.*, 1993; Toelner *et al.*, 1995). Periodic structures grown artificially are employed in which successive layers, being chemically and structurally equivalent, differ in that one layer is enriched with a Mössbauer isotope while an alternating layer contains the same chemical element with a natural isotopic abundance (Gusev *et al.*, 1993; Chumakov *et al.*, 1993). The interest in such structures is connected with prospects of their application in Mössbauer filtration of SR, providing monochromatization of SR to the degree of order 10–100Γ, where Γ is the frequency width of a Mössbauer line (Gusev *et al.*, 1993; Chumakov *et al.*, 1993; Toelner *et al.*, 1995).

The subject of this paper is the theoretical investigation of the Mössbauer surface-guided modes (MSGMs) in periodic media with a space modulation of the Mössbauer isotope concentration. The MSGMs are a direct analog of the well known surface-guided electromagnetic modes (SGEMs) in media with a space modulation of the dielectric constant (Yariv & Yeh, 1984; Belyakov *et al.*, 1992). However, the MSGMs have some specific features connected with the resonant character of the quanta interaction.

2. Conditions of total internal reflection of Mössbauer radiation

Let us consider the MSGM on a flat boundary between a vacuum and half-space with one-dimensional space modulation of the Mössbauer isotope concentration in the direction normal to the boundary (Fig. 1). The physical reason for the existence of both the SGEM and the MSGM is the total internal reflection (TIR) of radiation at the boundary together with its diffraction reflection within the periodic bulk medium (Yariv & Yeh, 1984; Belyakov *et al.*, 1992). At X-ray wavelengths, vacuum is an optically more dense medium than the periodical half-space, *i.e.* $\varepsilon_{\rm el} < 1$, where $\varepsilon_{\rm el}$ is the medium dielectric constant. With the Mössbauer isotope nuclei in the medium, in the narrow interval near the Mössbauer resonance frequency ω_r, the TIR conditions at the boundary in some frequency range can be fulfilled if the following condition holds,

$$\Delta\varepsilon_N + \varepsilon_{\rm el} > 1 \qquad (1)$$

where $\Delta\varepsilon_N$ is a contribution from the nuclear resonance interaction of quanta with Mössbauer nuclei. Presenting $\varepsilon_{\rm el}$ in the form $\varepsilon_{\rm el} = 1 - \Delta\varepsilon_{\rm el}$ and using the expression for $\Delta\varepsilon_N$ through f_o, the coherent amplitude of Mössbauer forward scattering, $\Delta\varepsilon_N = 4\pi f_o n/k^2$, the TIR condition (1) can be presented in the form

$${\rm Re}[(A\Gamma_i)(\omega_r - \omega + i\Gamma/2)^{-1}] > \Delta\varepsilon_{\rm el} \qquad (2)$$

where $A = fncN/\mathbf{k}^3$, f is the Lamb–Mössbauer factor, n is the density of Mössbauer isotope nuclei, c is the concentration of the Mössbauer isotope, $\mathbf{k}$ is the wavevector of Mössbauer quanta, N is a nuclear factor determined by parameters of the Mössbauer transition used, Γ and Γ_i are the total and radiation widths of the Mössbauer transition and ω is the frequency.

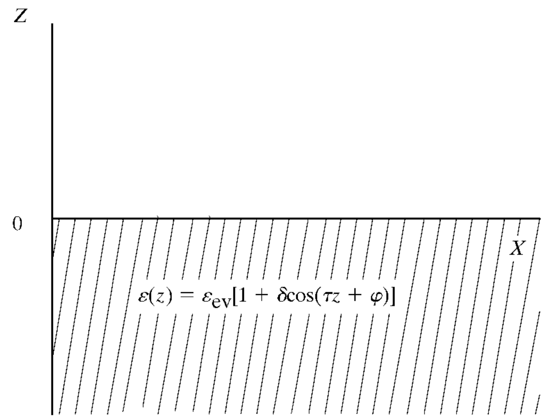

Figure 1
A description of the MSGM at the boundary of a vacuum and a half-space with a modulated Mössbauer isotope concentration.

From (2) it is obvious that the TIR condition can be fulfilled only for the frequencies which are lower than the resonant frequency ω_r. Assuming $\omega = \omega_r - \Delta\omega$, for the frequency range $\Delta\omega$ for which the TIR condition is fulfilled, we have

$$A\Gamma_1 \Delta\omega[(\Delta\omega)^2 + \Gamma^2/4]^{-1} > \Delta\varepsilon_{el}. \quad (3)$$

The necessary condition to satisfy (3) is that,

$$A\Gamma_1/\Gamma > \Delta\varepsilon_{el}$$

i.e.

$$c > (\Gamma/\Gamma_i)\Delta\varepsilon_{el}/(\mathrm{fnc}N/\mathbf{k}^3). \quad (4)$$

The condition (4) shows that there exists a critical value of the Mössbauer isotope concentration below which TIR is impossible. The estimates of the threshold concentration in the case of Fe give $c = 10\%$.

For concentrations larger than the critical one the TIR frequency interval is finite, reaches its maximum value at $c = 100\%$, and is determined by the condition,

$$d/2 - [(d/2)^2 - 1/4]^{1/2} < \Delta\omega/\Gamma < d/2 + [(d/2)^2 - 1/4]^{1/2} \quad (5)$$

where $d = A\Gamma_i/\Gamma\Delta\varepsilon_{el}$.

The estimate of the TIR frequency interval in metallic iron with a 100% concentration of Fe^{57}, for Mössbauer transition with the energy 14.4 keV, gives, according to (5), $\Delta\omega = 10\Gamma$.

3. Frequency region of MSGM existence

It is known (Belyakov *et al.*, 1992) that a threshold frequency ω_t for the SGEM exists. For the case presented here of the boundary with vacuum, this condition in the zero approximation of the amplitude of modulation of the Mössbauer isotope concentration δ takes the form

$$\omega \geq \omega_t = \tau c/2(\varepsilon_{ev}-1)^{1/2} = \tau c/2(\Delta\varepsilon_{Nev} - \Delta\varepsilon_{el}) \quad (6)$$

where τ is the reciprocal lattice vector, which determines the periodicity of the dielectric constant in the medium $\varepsilon(z) = \varepsilon_{ev}[1 + \delta\cos(\tau z + \varphi)]$, $\delta = \Delta\varepsilon_{Nev}\, m$ $(0 < m < 1)$, c is the velocity of light, ε_{ev} and $\Delta\varepsilon_{Nev}$ are averaged over the periodic medium total and the nuclear resonant parts of the dielectric constant. Taking $\Delta\varepsilon_{Nev}/2 = \Delta\varepsilon_{el}$ at the frequency of maximal $\mathrm{Re}(\Delta\varepsilon_{Nev})$ [see (1)], we find restrictions for $\Delta\omega$ compatible with (6),

$$\Delta/2 - [(\Delta/2)^2 - 1/4]^{1/2} < \Delta\omega/\Gamma < \Delta/2 + [(\Delta/2)^2 - 1/4]^{1/2} \quad (7)$$

where $\Delta = A(\Gamma_i/\Gamma)\{[\tau c/2(\omega_r - \Delta\omega)]^{1/2} + \Delta\varepsilon_{el}\}$. As $\Delta < d$ it follows from (5) and (7) that the allowed values of the MSGM frequency deviation from the exact resonance $\Delta\omega$ are determined with the region of overlap of (5) and (7). From (7) and Fig. 2 it follows that the MSGM existence concentration threshold exceeds by more than two the critical concentration of the Mössbauer isotope for the TIR threshold.

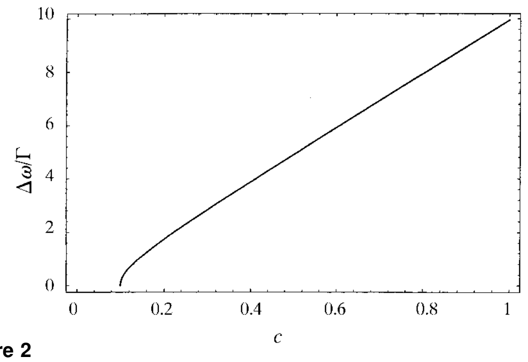

Figure 2
Concentration dependence of the frequency interval of TIR.

4. Dispersion equation

Supposing that the TIR conditions are fulfilled, we shall now look for solutions of MSGM with linear TE polarization in the form

$$E = C_1 \exp(iqx - \gamma_1 z),\ \gamma_1 = (q^2 - \omega^2/c^2)^{1/2},\ z > 0, \quad (8)$$

for the vacuum case, and in the form

$$\begin{aligned} E = B\{&\exp[i(\varphi+\beta)/2]\exp[-(\gamma + i\tau/2)z] \\ &+ \exp[-i(\varphi+\beta)/2]\exp[-(\gamma - i\tau/2)z]\}\exp(iqz),\ z<0 \end{aligned} \quad (9)$$

for the half-space case with a space modulation of the Mössbauer isotope concentration, where

$$\begin{aligned} &q = q_B + \Delta q,\ (q_B)^2 = (\kappa_0)^2 - \tau^2/4,\ (\kappa_0)^2 = \varepsilon_{ev}\omega^2/c^2, \\ &\Delta q = [(\delta/4)(\kappa_0)^2/q_B]\cos\beta, \\ &\gamma = [(\delta/2)(\kappa_0)^2/\tau]\sin\beta,\ -\pi<\beta<0. \end{aligned}$$

The dispersion equation for the MSGM is obtained from the continuity on the boundary at $z = 0$ tangent components of E and H,

$$\begin{aligned} &(\gamma_1 - \gamma)\cos[(\varphi+\beta)/2] + (\tau/2)\sin[(\varphi+\beta)/2] = 0. \\ &\gamma_1 = [\eta_1(\kappa_0)^2 - \tau^2/4 - (\delta/2)(\kappa_0)^2\cos\beta]^{1/2}. \end{aligned} \quad (10)$$

In the case of the MSGM parameters, γ and δ are complex quantities and δ is also a resonant function of frequency. All this complicates the analysis of equation (10) and strongly restricts the possibility of its analytical solution.

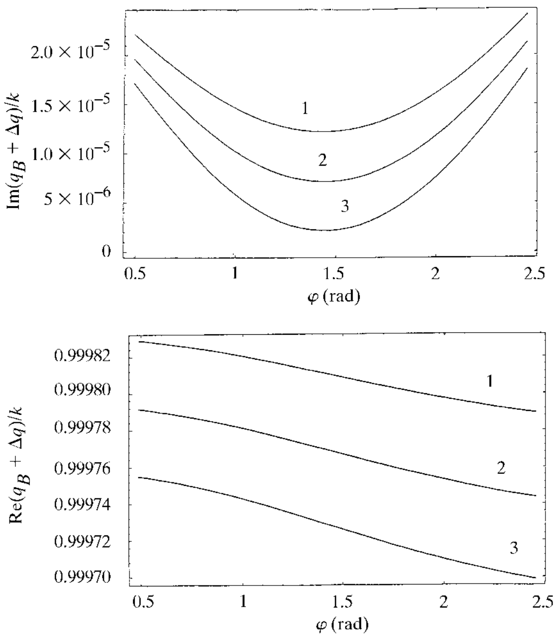

Figure 3
Solution of the dispersion equation: imaginary and real parts of the MSGM wavevector as a function of the phase of the dielectric constant modulation. (1) $\mathbf{k}/\tau = 24.5$, (2) $\mathbf{k}/\tau = 22.4$, (3) $\mathbf{k}/\tau = 20.7$; $\Delta\omega/\Gamma = 0.35$; $\delta = 0.1$.

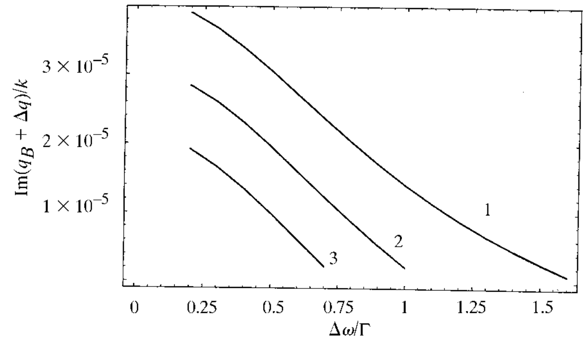

Figure 4
Solution of the dispersion equation: the spectral dependence of the imaginary part of MSGM wavevector. (1) $\mathbf{k}/\tau = 33.2$, (2) $\mathbf{k}/\tau = 24.5$, (3) $\mathbf{k}/\tau = 20.7$; $\varphi = 1.45$ rad; $\delta = 0.1$.

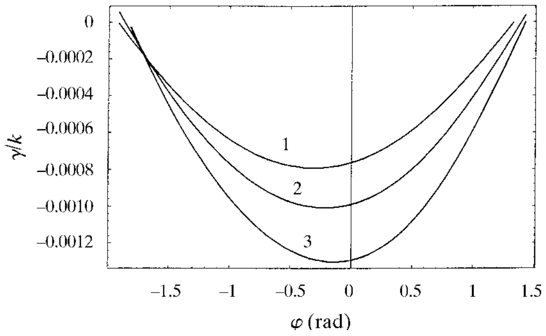

Figure 5
The dependence of the MSGM attenuation decrement inside the periodic medium *versus* the dielectric constant modulation phase. (1) $\mathbf{k}/\tau = 33.2$, (2) $\mathbf{k}/\tau = 26.5$, (3) $\mathbf{k}/\tau = 20.7$; $\Delta\omega/\Gamma = 0.35$; $\delta = 0.1$.

The results of numerical calculations are shown in Figs. 3–5. We hope that they will be useful for finding the optimal conditions for the experimental observation of MSGM.

5. Discussion and concluding remarks

One of the main problems facing the observation of the MSGM is its strong attenuation. Therefore, it seems to be very important to find conditions for which the MSGM attenuation is less than the attenuation of the Mössbauer radiation in the bulk of the medium. The results of the calculation give an opportunity to do this. As follows from Figs. 3 and 4 and from the general analysis, the attenuation of the MSGM decreases at $\mathrm{Re}\beta \simeq \pi/2$, which is reached at φ values approximately equal to $\pi/2$, and corresponds to the minimal penetration of radiation in the bulk of the strongly attenuating periodic medium.

Another cause for the decrease of the attenuation manifests itself near the threshold frequency. For the frequency precisely equal to that of the threshold there is no attenuation at all. In this case the main part of the MSGM energy is located in the vacuum. This is the cause of the drop of the attenuation close to the threshold frequency.

The minimal attenuation can be reached if both conditions mentioned above are satisfied. Roughly they look like

$$\lambda/2p = \theta_c, \quad \varphi \simeq \pi/2, \tag{11}$$

where θ_c is the TIR critical angle and p is the period of the Mössbauer isotope concentration modulation. Estimates according to formula (11) show that for iron the corresponding periods lie in the interval from 50 to 150 Å, *i.e.* exactly in the region of the periods of artificially grown isotopically enriched periodic structures (Chumakov *et al.*, 1992, 1993; Gusev *et al.*, 1993). Of course, more exact values of the parameters can be found by means of numerical calculations. As these calculations show, the attenuation of the MSGM propagating along the surface of a specially prepared medium with modulation of the Mössbauer isotope concentration can be essentially lower than the attenuation of Mössbauer quanta in the same medium, and even lower than the attenuation caused by an interaction of quanta with the electrons of the medium. All this opens up new prospects for the application of the MSGM for Mössbauer filtration of SR. However the first step towards the realization of this perspective still has to be performed, namely the experimental observation of the MSGM.

This work was supported by the RFBR grant No. 96-02-18812 and a grant within the Russian State Scientific Program 'Fundamental Metrology' on the subject of synchrotron radiation.

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