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Brilliance – an Overview

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One of the most important figures of merit for third-generation sources is the brilliance. An intuitive physical picture of the diffraction properties of synchrotron radiation is presented in this paper. The consequences of these diffraction properties on the design of third-generation light sources is also discussed.

Keywords: brilliance; third-generation sources; lattices; emittance; machine.

1. Introduction

The aim when designing synchrotron radiation rings is to match the ring properties to those of the emitted synchrotron radiation. During the evolution of synchrotron radiation sources, that is the magnet structures emitting synchrotron radiation, these goals have changed. For the first generation of synchrotron radiation rings, which mostly consist of high energy physics rings, dipole magnets are used as synchrotron radiation sources and the optimization is constrained by the existing design. The second generation of synchrotron radiation rings still use dipole magnets as main synchrotron radiation sources, but the electron (or positron) beam is optimized for synchrotron radiation uses.

Third-generation sources use special insertion devices, undulators and wigglers as synchrotron radiation sources, and these devices introduce new challenges in machine design.

We can only scratch slightly on the surface of machine physics in this paper. A more detailed description can be found in Courant & Snyder (1958), Wiedemann (1993), Jackson (1975) and Winick (1994). In this paper, a simple and intuitive picture of how accelerator physicists try to optimize storage rings to match the electron (positron) beams to the properties of light is given.

2. Figures of merit

There are mainly two properties characterizing a synchrotron radiation source. The first is the flux defined as

$$\Phi = \frac{N_f}{0.1\%,\,\mathrm{mrad}}$$

where Φ is in photons s^{-1} (0.1% bandwidth)$^{-1}$ (mrad horizontally)$^{-1}$ and N_f is the number of photons emitted s^{-1}.

The brilliance B is the peak flux density in phase space

$$B = \frac{N_f}{0.1\%,\,\mathrm{mm}^2\,\mathrm{mrad}^2}.$$

The flux is a function of electron current and electron energy only. When calculating the brilliance we have to take into account the phase space defined by the diffraction properties and that given by the electron-beam emittance.

Other figures of merit for a synchrotron radiation ring are electron-beam lifetime, spatial stability, ring reliability and ease of operation.

3. Specification goals

3.1. *Spectral region*

For a dipole source, the critical wavelength where intensity is close to maximum is given by $\lambda_c = 18.6/E^2B$ (Å), where E is the electron energy in GeV and B the magnetic field in T.

We should thus choose the electron energy to cover the photon spectral region of interest, but an electron energy that is too high might induce unnecessary heat problems and a ring cost that is too high. We must also take into account the fact that a large bending magnet radius will put the optical elements further away, which will reduce the opening angle for a given optical element size.

The use of insertion devices and then especially undulators puts even tighter boundary conditions on the electron energy. The spectral region is given by the undulator period and its magnet field. The total flux is proportional to the number of undulator periods. The latter is strongly dependent on the minimum undulator gap allowed in the machine.

As a consequence of this, the optimization process of a third-generation storage ring for synchrotron radiation is heavily dependent on the evolution of the insertion devices. A ring optimized for an undulator minimum gap of 40 mm, which was a conservative value some years ago, is certainly quite different from a ring optimized for a 10 mm gap, which is used and discussed today.

3.2. *Matching*

From Fig. 1, we can see that an observer will see the light emitted from a thin line source as originating from a shining disk, the apparent size of which is given by the diffraction relation. The line source could be a bending magnet which gives a short line source, or a longer undulator. The photon angular spread defined by the diffraction can generally be approximated for Gaussian distributions to $\sigma'_{ph} = (\lambda/2L)^{1/2}$ where σ' is the r.m.s. angular spread, λ is the emitted light wavelength and L the source length.

The diffraction relation for Gaussian distributions $\sigma_{ph}\sigma'_{ph} = \lambda/4\pi$ yields the apparent source size $\sigma_{ph} = (1/\pi)(\lambda L/8)^{1/2}$.

The electron-beam phase-space area or emittance $\varepsilon = \sigma_e\sigma'_e$ should thus preferably be less than $\lambda/4\pi$. If this is the case, the source is called diffraction-limited. This demand is so far difficult to fulfil, especially for shorter wavelengths.

Even if the electron-beam emittance is smaller than $\lambda/4\pi$, we need to match the phase-space form of the electron-beam emittance to that induced by diffraction. The mismatch between these phase-space forms for a dipole source is seen in Fig. 2, where L is in the mm range which gives σ_{ph} in the μs range while the electron-beam size is orders of magnitude larger.

For an undulator, with L in the meter region, σ_{ph} is some order of magnitude larger that yields a better match to the electron-beam size. This fact plus the larger number of emitting sources explains the high brilliance attainable from an undulator in a low-emittance storage ring.

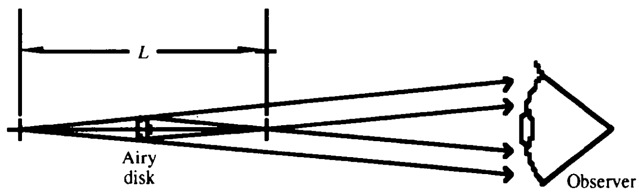

Figure 1
Light from a long light source.

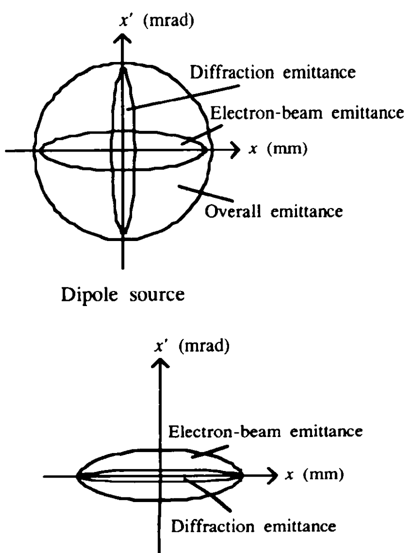

Figure 2
Photon and electron-beam emittances in phase space.

4. Magnet lattice

4.1. *Betatron motion*

A ring magnet lattice consists mainly of dipole magnets which recirculate the electron beam and magnet lenses, quadrupole magnets. These lenses will force the stored particles to oscillate around a reference orbit. If we smear out the focusing evenly around the ring, we should obviously get a harmonic oscillation for a certain particle

$$x(\Psi) = C\sin(K\Psi + \Phi)$$

where C is the oscillation amplitude, K is the wavenumber, Ψ the azimuth angle in the ring and Φ a phase angle.

In a real machine, we have piecewise constant focusing lenses. A solution to the particle motion can then be described as

$$x(s) = C[\beta(s)]^{1/2}\sin[Q\Psi(s) + \Phi].$$

$\beta(s)$ is now a modulating function, s the coordinate in the reference-orbit direction and Q is the betatron wavenumber defining the number of oscillations the particle will execute during one turn in the machine. Ψ is now not exactly the azimuth angle but is generally close to it.

The angle between the particle motion and the reference orbit is achieved by differentiating x with respect to s

$$x'(s) = \mathrm{d}x(s)/\mathrm{d}s.$$

In phase space xx' the particle will now describe an ellipse when moving in the ring. The oscillation amplitudes for the particles in the beam will have a Gaussian distribution. The particle with the r.m.s. amplitude will have its motion described by

$$x(s) = [\varepsilon\beta(s)]^{1/2}\sin[Q\Psi(s) + \Phi]$$

where ε defining the oscillation amplitude is the area (divided by π) of the phase-space ellipse and is called the electron-beam emittance.

We have two directions of oscillations, radial and vertical, so we now have two β functions.

4.2. *Dispersion*

We see in Fig. 3 the basic building block of a storage ring. The electron beam enters from the left. Particles of higher energies are bent less than those of nominal energy, but are focused back by the middle lens.

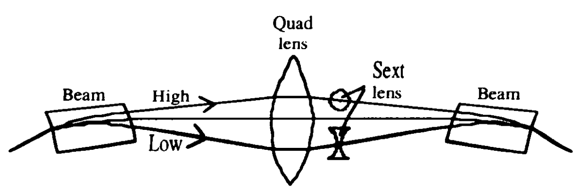

Figure 3
Double-bend achromat.

The function, describing the position of an off-energy particle with a momentum deviation $\Delta P/P = 1$, is called the dispersion function η and this is also seen in Fig. 3.

5. Radiation damping and beam emittance

As we have seen above, the particles stored in the ring execute oscillations around a reference orbit. Let us first look for the radiation damping in the vertical direction as seen in Fig. 4.

An oscillating particle will emit a photon in a dipole magnet. The recoil of the photon will have one longitudinal component retarding the particle and one vertical component damping the vertical oscillation. The longitudinal recoil will be replaced by the accelerating cavity while the vertical recoil component will damp the oscillation.

The situation is somewhat more complicated in the radial direction. Let us look at Fig. 4 where we assume that a particle of nominal energy on the reference orbit emits a photon in a bending magnet. Apart from the damping mechanism described for the vertical case above, the particle will now suddenly have another equilibrium orbit introduced by its changed energy and the present non-zero dispersion. The introduced oscillation amplitude equals the particle distance to the new equilibrium orbit.

This mechanism is in fact responsible for the equilibrium emittance in a storage ring. We can also see that a large dispersion in the bending magnets introduces large oscillation amplitudes and thus a larger emittance.

6. Decreasing the electron-beam emittance

The chase is now on for smaller dispersion functions in the bending magnets in order to decrease the particle-beam emittance and thus increase the brilliance. The first step is of course to enlarge the number of achromats to obtain a small dispersion in the bending magnets. The smaller the bending angle of the bending magnet, the smaller the dispersion. We can also lower the electron-beam energy and get a smaller recoil. The emittance scales in fact like

$$\varepsilon E^2/N^3$$

where E is the electron energy and N the number of bending magnets.

We can, however, not push this scheme too far. The reason for this is chromaticity corrections. A particle of higher energy will experience a weaker focusing in the quadrupole lenses and should thus have a lower betatron wavenumber Q. The chromaticity

$$\xi = \mathrm{d}Q/(\mathrm{d}P/P)$$

will then be negative which will excite head–tail instability. We correct the chromaticity by introducing nonlinear lenses, sextupole magnets, at positions with non-zero dispersion as seen in Fig. 3. The particle position is here energy dependent and the nonlinear sextupole lens can then compensate the betatron Q value.

For small dispersion functions, we need stronger sextupoles and this will make the lattice more unstable, as the dynamic aperture is reduced. This mechanism is one factor limiting the possibility of reaching small remittances.

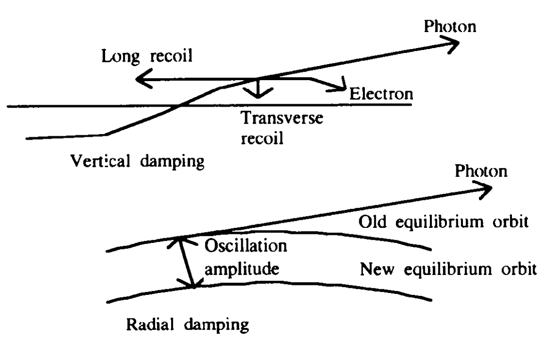

Figure 4
Vertical and radial damping.

7. Performance limitations

Even if in principle we know the way to diffraction-limited emittances, there are a number of problems blocking this approach.

One is money. Decreasing the particle emittance means a larger number of magnets are required to decrease the dispersion. The machines tend to be big and costly.

A second is dynamic stability. As we increase the sextupole strengths, we reduce the dynamic aperture. This causes a reduction of beam lifetime and increases the alignment problems.

A third is the possibility of effectively keeping the beam position inside a fraction of the beam size. As we reduce the beam size, we have to position the beam more exactly. If the experiments should be able to benefit from the smaller beam size, then the effective emittance seen by the user should not be diluted by beam motions.

We are also aiming at high mean currents. This means high maximum current and a lifetime that is long compared with the injection time.

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