steepest-descents method to $R$ is equivalent to a method based on a Fourier difference synthesis.

$$
\begin{equation*}
D=\sum_{h k l}\left(F_{o}-F_{c}\right) \cos 2 \pi(h x+k y+l z) \tag{2}
\end{equation*}
$$

In this series, $x, y, z$ are the fractional co-ordinates, taken over one unit cell.

The method consists of drawing a map of the function $D$ and marking the atomic positions $x_{i}, y_{i}, z_{i}(i=1,2, \ldots, N)$. The atomic shifts $\Delta x_{i}, \Delta y_{i}, \Delta z_{i}$, which would improve the structure according to the method of steepest descents, can be obtained by moving the atomic positions in the directions of the steepest ascent of $D$ and by making the movements proportional to the gradient of $D$ at the points $x_{i}, y_{i}, z_{i}$. Good use of this method in practical application has been made by Cochran (private communication).

As Booth (1949) has already pointed out, considerable simplifications in the steepest-descents calculations can be achieved if one takes

$$
\begin{equation*}
R=\sum_{h k l}\left|F_{o}-F_{c}\right| \tag{3}
\end{equation*}
$$

This function can be obtained from ( $1 a$ ) by taking for the weight

$$
W=\frac{1}{\left|F_{o}-F_{c}\right|}
$$

This form of weighting is not quite correct, as it gives too little weight to large discrepancies between the observed and calculated values. However, the function (3) is often used as a basis for the calculation of the figure of merit.

As the use of the difference synthesis is very convenient, it is of interest to investigate whether an analogous simpler function exists, which would be based on (3). For this purpose we can write $R$ in the following form:

$$
\begin{equation*}
R=\Sigma\left(\alpha\left|F_{o}\right|-\beta F_{c}\right)=\Sigma \alpha\left|F_{o}\right|-\Sigma \beta F_{c} \tag{4}
\end{equation*}
$$

where the coefficients $\alpha(h k l)$ and $\beta(h k l)$ have the values

$$
\begin{gathered}
\begin{array}{c}
\alpha=+1 \text { for }\left|F_{c}\right|<\left|F_{o}\right|, \quad \alpha=-1 \text { for }\left|F_{c}\right|>\left|F_{o}\right|, \\
\beta=+1 \text { for }\left|F_{o}\right|>F_{c}>0 \text { and } F_{c}<-\left|F_{o}\right|, \\
\beta=-1 \text { for } 0>F_{c}>-\left|F_{o}\right| \text { and } F_{c}>\left|F_{o}\right|, \\
\text { i.e. } \quad \beta=+1 \text { for }\left(F_{o}-F_{c}\right)>0, \\
\beta=-1 \text { for }\left(F_{o}-F_{c}\right)<0 .
\end{array}
\end{gathered}
$$

Limiting ourselves to a centrosymmetrical structure, we can substitute for $F_{c}$ in (4):

$$
\begin{equation*}
F_{c}=2 \sum_{i}^{N} f_{i} \cos 2 \pi\left(h x_{i}+k y_{i}+l z_{i}\right) \tag{5}
\end{equation*}
$$

where $N$ is the number of atoms per half unit cell. We obtain

$$
R=\sum_{h k l} \alpha\left|F_{o}\right|-2 \sum_{h k l} \beta \sum_{i} f_{i} \cos 2 \pi\left(h x_{i}+k y_{i}+l z_{i}\right)
$$

On reversing the order of summation, we can write

$$
\begin{equation*}
R=R_{0}-\sum_{i} R_{i} \tag{6}
\end{equation*}
$$

where

$$
\left.\begin{array}{l}
R_{0}=\sum_{h k l} \alpha\left|F_{o}\right|  \tag{7}\\
R_{i}=2 \sum_{h k l} \beta f_{i} \cos 2 \pi\left(h x_{i}+k y_{i}+l z_{i}\right) \cdot
\end{array}\right\}
$$

We have achieved separation of $R$ into $R_{0}$, which depends on $F_{0}$ only, and into $N$ functions $R_{i}$, each of which depends on the co-ordinates of the $i$ th atom only. In addition, $R_{i}$ have the form of a Fourier synthesis analogous to the difference synthesis (2), i.e. if we replace in (7) the discrete co-ordinates $x_{i}, y_{i}, z_{i}$, by continuous $x, y, z$, a single function is obtained, defined over the whole unit cell, which has the values $R_{i}$ at the atomic positions. The synthesis (7) differs from (2) in that its coefficients have absolute values independent of $F_{o}$ and $F_{c}$; only their signs depend on the inequalities for $\beta$ and are incidentally the same as in (2). It should be noted, however, that although $R$ is a continuous function of the atomic co-ordinates $x_{i}, y_{i}, z_{i}$, the functions $R_{0}$ and $R_{i}$ are discontinuous, the discontinuities occurring every time the coefficient $\beta$ changes sign. One can visualize this by imagining $R$ as a function of the co-ordinates $x_{i}, y_{i}, z_{i}$, in $3 N$-dimensional space, which is subdivided into a number of small cells, the boundaries of which are given by the changes of sign of $\beta$. Within each of the cells, $R_{0}$ and $R_{i}$ are continuous functions of the co-ordinates. The number of such cells depends on the number of $F$ terms included in the summations.

As formula (7) is based on incorrect weighting, it is to be expected that the above method would converge more slowly than a method based on (2), and therefore more steps would be required to attain a given degree of accuracy. However, the use of the series (7) may in some cases have the advantage that its coefficients are fixed quantities, and only their signs change during calculation. The series (7) might lend itself therefore better to calculations on automatic machines, analogue or digital, as it is easier in a calculation to change the sign of a term than its value. The inequalities for $\alpha$ and $\beta$ lend themselves also admirably for automatic evaluation on both digital and analogue machines. One can therefore conclude that the calculations based on formula (3) might be more suitable for a fully or partly automatic cycle, especially where the simplicity of setting or programming is of more importance than the speed of convergence per cycle.

## References

Boort, A. D. (1947). Nature, Lond., 160, 196.
Воотн, A. D. (1948). Nature, Lond., 161, 765.
Воотн, A. D. (1949). Proc. Roy. Soc. A, 197, 336.

## Notes and News

Announcements and other items of crystallographic interest will be published under this heading at the discretion of the Editorial Board. Copy should be sent direct to the British Co-editor (R. C. Evans, Crystallographic Laboratory, Cavendish Laboratory, Cambridge, England).

## $X$-ray Identification and Crystal Structures of

 Clay MineralsThe Mineralogical Society of Great Britain announces the publication of a new and original monograph of 345 pages. The preparation of this book results from the efforts and
discussions of the Clay Minerals Group which was formed in 1947 for the express purpose of furthering the study of clay minerals.

The book provides an authoritative account of the X-ray identification and crystal structures of clays and
allied substances. The fourteen chapters are contributed by a number of different authors who are experts in their respective fields, the co-ordinating editor being Dr G. W. Brindley. The kaolin, montmorillonite, mica and chlorite minerals, vermiculite, sepiolite, attapulgite and oxides of iron and aluminium are all included in the survey. A great deal of information is presented in the form of tables, valuable, for reference purposes, not only to those concerned with the study and applications of clays, but also to X-ray crystallographers generally.

Copies (price $35 s$.; post free $36 s$.) are obtainable only from the General Secretary, Mineralogical Society, British Museum (Natural History), Cromwell Road, London S.W.7, England or in the U.S.A. (price \$6), from Dr R. E. Grim, Illinois Geological Survey, Urbana, Illinois, U.S.A.

## International Union of Crystallography

Notification of adhesion from 1 January 1951 has been given by Sweden through the Swedish National Com-
mittee for Crystallography and by Italy through the
National Research Council. The Adhering Bodies are now :

| Australia | Group I |
| :--- | :--- |
| Belgium | Group III |
| Canada | Group IV |
| Czechoslovakia | Group I |
| Denmark | Group I |
| France | Group VII |
| India | Group I |
| Italy | Group III |
| Japan | Group I |
| Netherlands | Group IV |
| Norway | Group I |
| South Africa | Group I |
| Spain | Group IV |
| Sweden | Group I |
| Switzerland | Group I |
| United Kingdom | Group VIII |
| United States of America | Group VIII |

## Books Received

The undermentioned works have been received by the Editors. Mention here docs not proclvde review at a later date.

X-ray Identification and Crystal Structures of Clay Minerals. Edited by G. W. Brindley. Obtainable only from the General Secretary, Mineralogical Society, British Museum (Natural History), London S.W. 7, England; or Dr R. E. Grim, Illinois Geological Survey, Urbana, Illinois, U.S.A. Pp. 345 with many figures and tables.

London: Mineralogical Society (Clay Minerals Group). 1951. Price 35s.; \$6.

Geometrische Kristallographie und Kristalloptik. By F. Raaz and H. Tertsch. Pp. x+215, with 260 figs. Vienna: Springer. 2nd ed. 1951. Price 32s. 6 d .

## International Union of Crystallography

The audited accounts of the Union for the calendar year 1950 are printed below.
Acta Crystallographica Account for the year ended 31 December 1950


International Tables Account for the year ended 31 December 1950

