

steepest-descents method to R is equivalent to a method based on a Fourier difference synthesis.

$$D = \sum_{hkl} (F_o - F_c) \cos 2\pi(hx + ky + lz). \quad (2)$$

In this series, x, y, z are the fractional co-ordinates, taken over one unit cell.

The method consists of drawing a map of the function D and marking the atomic positions x_i, y_i, z_i ($i = 1, 2, \dots, N$). The atomic shifts $\Delta x_i, \Delta y_i, \Delta z_i$, which would improve the structure according to the method of steepest descents, can be obtained by moving the atomic positions in the directions of the steepest ascent of D and by making the movements proportional to the gradient of D at the points x_i, y_i, z_i . Good use of this method in practical application has been made by Cochran (private communication).

As Booth (1949) has already pointed out, considerable simplifications in the steepest-descents calculations can be achieved if one takes

$$R = \sum_{hkl} |F_o - F_c|. \quad (3)$$

This function can be obtained from (1a) by taking for the weight

$$W = \frac{1}{|F_o - F_c|}.$$

This form of weighting is not quite correct, as it gives too little weight to large discrepancies between the observed and calculated values. However, the function (3) is often used as a basis for the calculation of the figure of merit.

As the use of the difference synthesis is very convenient, it is of interest to investigate whether an analogous simpler function exists, which would be based on (3). For this purpose we can write R in the following form:

$$R = \sum (\alpha |F_o| - \beta F_c) = \sum \alpha |F_o| - \sum \beta F_c, \quad (4)$$

where the coefficients $\alpha(hkl)$ and $\beta(hkl)$ have the values

$$\alpha = +1 \text{ for } |F_c| < |F_o|, \quad \alpha = -1 \text{ for } |F_c| > |F_o|,$$

$$\beta = +1 \text{ for } |F_o| > F_c > 0 \text{ and } F_c < -|F_o|,$$

$$\beta = -1 \text{ for } 0 > F_c > -|F_o| \text{ and } F_c > |F_o|,$$

$$\text{i.e. } \beta = +1 \text{ for } (F_o - F_c) > 0,$$

$$\beta = -1 \text{ for } (F_o - F_c) < 0.$$

Limiting ourselves to a centrosymmetrical structure, we can substitute for F_c in (4):

$$F_c = 2 \sum_i^N f_i \cos 2\pi(hx_i + ky_i + lz_i), \quad (5)$$

where N is the number of atoms per half unit cell. We obtain

$$R = \sum_{hkl} \alpha |F_o| - 2 \sum_{hkl} \beta \sum_i f_i \cos 2\pi(hx_i + ky_i + lz_i).$$

On reversing the order of summation, we can write

$$R = R_0 - \sum_i R_i, \quad (6)$$

$$\left. \begin{aligned} \text{where } R_0 &= \sum_{hkl} \alpha |F_o|, \\ R_i &= 2 \sum_{hkl} \beta f_i \cos 2\pi(hx_i + ky_i + lz_i). \end{aligned} \right\} \quad (7)$$

We have achieved separation of R into R_0 , which depends on F_o only, and into N functions R_i , each of which depends on the co-ordinates of the i th atom only. In addition, R_i have the form of a Fourier synthesis analogous to the difference synthesis (2), i.e. if we replace in (7) the discrete co-ordinates x_i, y_i, z_i , by continuous x, y, z , a single function is obtained, defined over the whole unit cell, which has the values R_i at the atomic positions. The synthesis (7) differs from (2) in that its coefficients have absolute values independent of F_o and F_c ; only their signs depend on the inequalities for β and are incidentally the same as in (2). It should be noted, however, that although R is a continuous function of the atomic co-ordinates x_i, y_i, z_i , the functions R_0 and R_i are discontinuous, the discontinuities occurring every time the coefficient β changes sign. One can visualize this by imagining R as a function of the co-ordinates x_i, y_i, z_i , in $3N$ -dimensional space, which is subdivided into a number of small cells, the boundaries of which are given by the changes of sign of β . Within each of the cells, R_0 and R_i are continuous functions of the co-ordinates. The number of such cells depends on the number of F terms included in the summations.

As formula (7) is based on incorrect weighting, it is to be expected that the above method would converge more slowly than a method based on (2), and therefore more steps would be required to attain a given degree of accuracy. However, the use of the series (7) may in some cases have the advantage that its coefficients are fixed quantities, and only their signs change during calculation. The series (7) might lend itself therefore better to calculations on automatic machines, analogue or digital, as it is easier in a calculation to change the sign of a term than its value. The inequalities for α and β lend themselves also admirably for automatic evaluation on both digital and analogue machines. One can therefore conclude that the calculations based on formula (3) might be more suitable for a fully or partly automatic cycle, especially where the simplicity of setting or programming is of more importance than the speed of convergence per cycle.

References

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