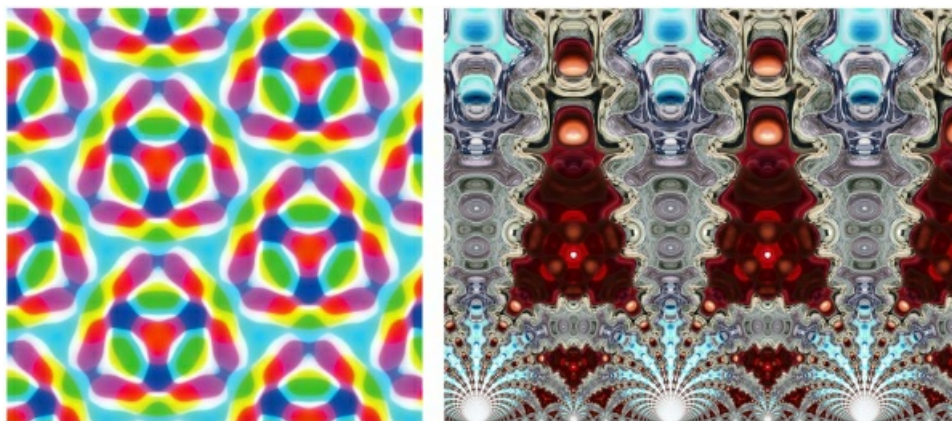


*Quantitative classification of periodic gray-level patterns by geometric AIC*Peter Moeck¹¹Portland State University, Portland, United States

E-mail: pmoeck@pdx.edu

An information theoretical approach to monochrome digitized more or less 2D periodic images with high to moderate signal to noise ratios coupled with geometric inferences (on the basis of geometric Akaike Information Criteria) allows for unambiguous (quantitative) detections of the plane symmetries and Bravais lattices [1] that the underlying self-assembled 2D periodic arrays of atoms or molecules (of strictly the same kind and mutual time averaged arrangement) possess. While these images are necessarily of finite size, the information they contain is either from the whole array or represented as convolution of an aperture function with the array. (The mathematically strict requirement of 2D periodicity out to infinity is relaxed for the sake of the practicality of the classification.) This approach is in principle independent of the type of employed data recording device and has been utilized in the computational symmetry [2] and scanning probe microscopy [1] communities. The image processing and geometric inferences can be done either in direct [2] or reciprocal (Fourier) [1] space. For a large number of unit cells and large sizes of data files, the reciprocal space approach is computationally much more effective and allows for the usage of core features of existing computer programs (https://en.wikipedia.org/wiki/Crystallographic_image_processing) as pioneered by structural biologists and electron crystallographers. While there is an analogous quantitative classification approach in 1D periodic space [2], extensions to 3D periodic gray-level patterns are not trivial. Within the framework of a quantitative crystallographic classification of periodic ornaments, building decorations, ... and works of art from digitized gray-scale images, one could consider the plane symmetry enforced version of the data as "signal", which is in the information theory context minimized relative Kullback-Leibler divergence. Both intended and unintended breakings of plane symmetries by the artist or artisan could formally be classified as "noise" (residuals) or variation left unexplained, which may make works of art and cultural artefacts all the more appealing to members of the subspecies homo sapiens sapiens. Quantitative crystallographic classifications are demonstrated on scanning probe microscope images, cultural artifacts of the "computer age", e.g. Figures (<http://www.pnas.org/content/112/45/13747.full#F1> and ...#F2) below, as programmed by the mathematician-artist Frank A. Farris [3], and digitized works of art by Maurits Cornelis Escher.

[1] Straton, J. C. et al. (2014). *Cryst. Res. Technol.* 49, 663–680. [2] Liu, Y. et al. (2004). *IEEE Trans. Pattern Anal. Machine Intel.* 26 354–371. [3] Farris, F. A. (2015). *Creating Symmetry, The artful mathematics of wallpaper patterns*, Princeton University Press.



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