Poster Presentation

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Symmetry Groups Associated with Tilings of a Flat Torus

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A flat torus E^2/Λ is the quotient of the Euclidean plane E^2 with a full rank lattice Λ generated by two linearly independent vectors v_1 and v_2. A motif-transitive tiling T of the plane whose symmetry group G contains translations with vectors v_1 and v_2 induces a tiling T^* of the flat torus. Using a sequence of injective maps, we realize T^* as a tiling $\bar{T}$ of a round torus (the surface of a doughnut) in the Euclidean space E^3. This realization is obtained by embedding T^* into the Clifford torus $S^1 \times S^1 \subseteq E^4$ and then stereographically projecting its image to E^3. We then associate two groups of isometries with the tiling T^* – the symmetry group G^* of T^* itself and the symmetry group $\bar{G}$ of its Euclidean realization $\bar{T}$. This work provides a method to compute for G^* and $\bar{G}$ using results from the theory of space forms, abstract polytopes, and transformation geometry. Furthermore, we present results on the color symmetry properties of the toroidal tiling T^* in relation with the color symmetry properties of the planar tiling T. As an application, we construct toroidal polyhedra from $\bar{T}$ and use these geometric structures to model carbon nanotori and their structural analogs.

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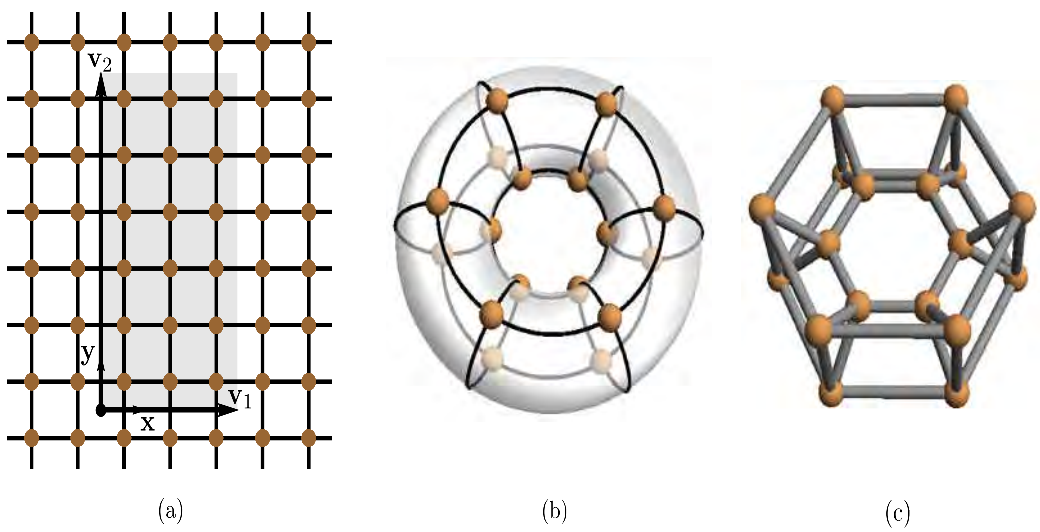

Figure. (a) A 4^4 tiling $\mathbb{T}$ of the plane $\mathbb{E}^2$ with translation vectors **x**, **y** and orthogonal vectors $\mathbf{v}_1 = 3\mathbf{x}$, $\mathbf{v}_2 = 6\mathbf{y}$ generating a lattice Λ. (b) Geometric realization of the tiling $\mathbb{T}^*$ of the flat torus $\mathbb{E}^2/\Lambda$ as a tiling $\overline{\mathbb{T}}$ of a round torus. (c) A toroidal polyhedron obtained from $\overline{\mathbb{T}}$ in (b) by replacing the edges of the tiling with straight segments.

Keywords: Flat Torus, Tiling, Nanotorus