## 20. SYMMETRY AND ITS GENERALIZATION

where $G_{1}$ is generated by a $3_{1}$ screw rotation, $C_{2}$ is generated by plane reflections in the side planes of a regular trigonal prisma and hence $C_{2}$ is isomorphic to the plane crystallographic group above. Combining these, we get a presentation
$R 3 m=\left(m, s-1=m^{2}=s^{-3} m s^{3} m=\left(s^{-I_{m s m}}\right)^{3}\right)$
belonging to a concave topological polyhedron F. This polyhedron $F$ has only three faces. The face $f_{m}$ corresponds to the plane reflection $m: f_{m} \rightarrow f_{m}$ and the curved faces $f_{s^{-1}}$ $f_{s}$ are identified by the $3_{1}$ screw-rotation $S^{s}: f_{S^{-1}} \rightarrow f_{s}$ with screw-rotation angle $\frac{2 \pi}{3}$. The presentation is minimal, i.e. $F$ has the minimum number of faces. $F$ is a topological polyhedron, i.e. the body of $F$ is homeomorphic to a 3-dimensional simplex, each face of $F$ to a $2-s i m p l e x$ and so on.

This geometric presentation of the described space groups, illustrated also by Figures, can give a more complete information on the structure of each group.
20.2-8 ON CONSISTENT SETS OF ASYMMETRIC UNITS. By W. Fischer, Institut für Mineralogie, PhilippsUniverstät, 3550 Marburg, FRG
An asymmetric unit of a space group $G$ is a smallest part of 3-dimensional space from which the entire space may be generated by the action of $G$. Therefore, all inner points of an asymmetric unit are symmetrically inequivalent to each other with respect to G. Different definitions have been used so far concerning points on the boundaries. Normally, an asymmetric unit is supposed to be simply connected and convex, then it is a polyhedron. These additional conditions can always be fulfilled, because the asymmetric unit may be constructed as Dirichlet domain of a point out of any general point configuration of $G$. In this case, adjacent asymmetric units share entire faces (the corresponding space tiling is called normal), but the polyhedron may be unnecessarily complicated in shape.
Two sets of asymmetric units have been published, one for all space groups by $H$. Arnold (in: International Tables for Crystallography, Vol. A (1983), D. ReideI), the other only for cubic ones by E. Koch \& W. Fischer (Acta Cryst. (1974), A30, 490). Arnold's set is chosen in such a way, that Fourier summation can be performed conveniently. It contains asymmetric units with non-normal space tilings (cf.e.g. P4 ). This is not the case with the KochFischer set which is derived from Dirichlet domains and uses polyhedra with minimal numbers of faces. Both sets do not take care of group-subgroup relations.
For comparative studies (e.g. of relations between crystal structures) sets of asymmetric units would be useful where the asymmetric unit of any space group $G$ is composed of entire asymmetric units of any supergroup $H \supset G$. This, however, seems unachievadie because of the complexity of subgroup relations between space groups. Especially for studies of geometrical properties (for a
list of references cf. W. Fischer \& E. Koch, Acta Cryst. (1983), A39, 907), however, a less severe restriction is helpful: A set of asymmetric units will be called consistent, if the asymmetric unit of any $G$ is composed of entire asymmetric units of the Euclidean normalizer (Cheshire group) $N_{E}(G)$. The Euclidean normalizers of space groups belong to $E_{30}$ types either of space groups or of their degenerations with continuous translations (F.L. Hirshfeld, Acta Cryst. (1968), A24, 301). As a space group occuring as Euclidean normalizer may itself have a Euclidean normalizer of another type, consistent sets of asymmetric units have to be based on a suitable choice for a smaller number of sumnits ( $1 \mathrm{~m}^{3} \mathrm{~m}$, Ia Z d , $\mathrm{Pb} / \mathrm{mmm}, \mathrm{Pb}_{2} 22-\mathrm{Pb}_{4}^{22}, \mathrm{R} 3 \mathrm{~m}, \mathrm{P} 4 / \mathrm{mmm}, \mathrm{Pmmm}, \mathrm{P} 2 / \mathrm{m}, \mathrm{P} 1$; $\left.z^{1} 6 / \mathrm{mmm}, \quad z^{1} 4 / \mathrm{mmm}, z^{1} \mathrm{mmm}, z^{1} 2 / \mathrm{m}, z^{2} 2 / \mathrm{m}, z^{3} \overline{1}\right)$.

Outside the cubic crystal system Arnold's set differs from a consistent one only for space groups $\mathrm{P} 2 / \mathrm{m}$, I4 $/ \mathrm{A}$, $P 4_{2} 22, R \overline{3}, P 31_{1} 12-P 3_{2} 12, R 32$ and $R \overline{3} C$. Within the cubic system both published sets are far from being consistent. Summit Im3̃ poses no problems: the unique asymmetric unit of Pm3m may be subdivided by a plane containing the twofold axis at $1 / 2-x, 1 / 4, x$. Consistent sets for cubic space groups other than la $\overline{3} d$ and its subgroups result if this plane is chosen either at $x+z=1 / 2$ (case 1) or at $y=1 / 4$ (case 2). In both cases, the asymmetric units of some space groups may be selected in different ways. only in case 1 it is possible to restrict to normal space tilings. The number of differently shaped asymmetric units is smaller for case 1 than for case 2. Two other specialized positions of the subdividing plane, i.e. $x-2 y+z=0$ (case 3) and $x+y+z=3 / 4$ (case 4), do not give rise to consistent sets, because the asymmetric units of $\mathrm{Fd} \overline{3} \mathrm{~m}, \mathrm{Fd} \overline{3}$, and $\mathrm{F} 4,32$ cannot be made convex. - For Ia $\overline{3} d$ and its subgroups apparently no consistent set of convex asymmetric units can be constructed, but the impossibility of such a set could not be proved so far.
20.3-1 THE POLYTYPES OF THE ORTHOROMBIC CARBIDE $M_{7} \mathrm{C}_{3}$. By M. Kowalski and WoDudziński, Institute of Material Science, Technical University of Wrocław, Poland.

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[^0]:    The stacking order of the atomic layers in the real crystals of the orthorombic carbide /Cr, Fe/7C3 was studed by means of the transmission electron microscopy. The orthorombic carbides of the type $M_{7} \mathrm{C}_{3}$ can be regarded as built up of identical layers of structure stacked parallel to (110) planes. The information about stacking order of the layers is contained in the intensity diatribution of diffraction spots observed along [110]" direction of the reciprocal lattice. In the real crystals regions with completly disordered structure /fig. $1 /$ and ordered sequence of the layers/fig. $2 /$ can be observed. In our earlier paper /XI-th Conference of Material Science, 1983, Częstochowa, Poland/we described the stacking order using the concept of the polytypism, and we presented the structure of the 20 polytype. Systematical study of the ordered regions in the/Cr, $\mathrm{Fe} / 7 \mathrm{C}_{3}$ carbides let us determine the structure of the other polytypes. Lattice parameters was determined by analisys of the geometry of dis tribution of the diffraction spots in the planes (hki) of the reciprocal lattice. The stacking sequence in the unit cell was identified by comparision of the observed intensity distribution of diffraction spots with the intensity calculated for theoritically assumed sequence of the layers. The polytypes found in studed carbides have a following crystalographic dates:

